MODEL SOLUTIONS

KEY

GCSE Mathematics

Practice Tests: Set 9

Paper 1H (Non-calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Area of 2D Shapes

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Here is a hexagon *ABCDEF*.

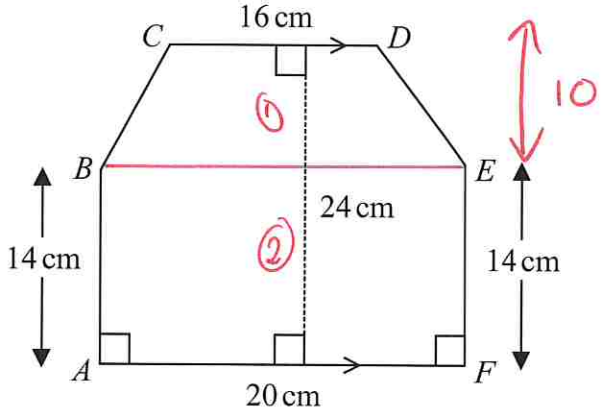


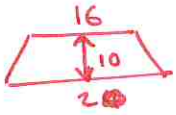
Diagram **NOT** accurately drawn

CD is parallel to *AF*.

Work out the area of hexagon *ABCDEF*.

① Area of trapezium

$A = \frac{h(a+b)}{2}$



$A = \frac{10(16+20)}{2} = \frac{10(36)}{2} = 180\ cm^2$

② Area of rectangle

$A = L \times W$

$A = 20 \times 14 = 280\ cm^2$

Total Area

① + ② $= 180\ cm^2 + 280\ cm^2$

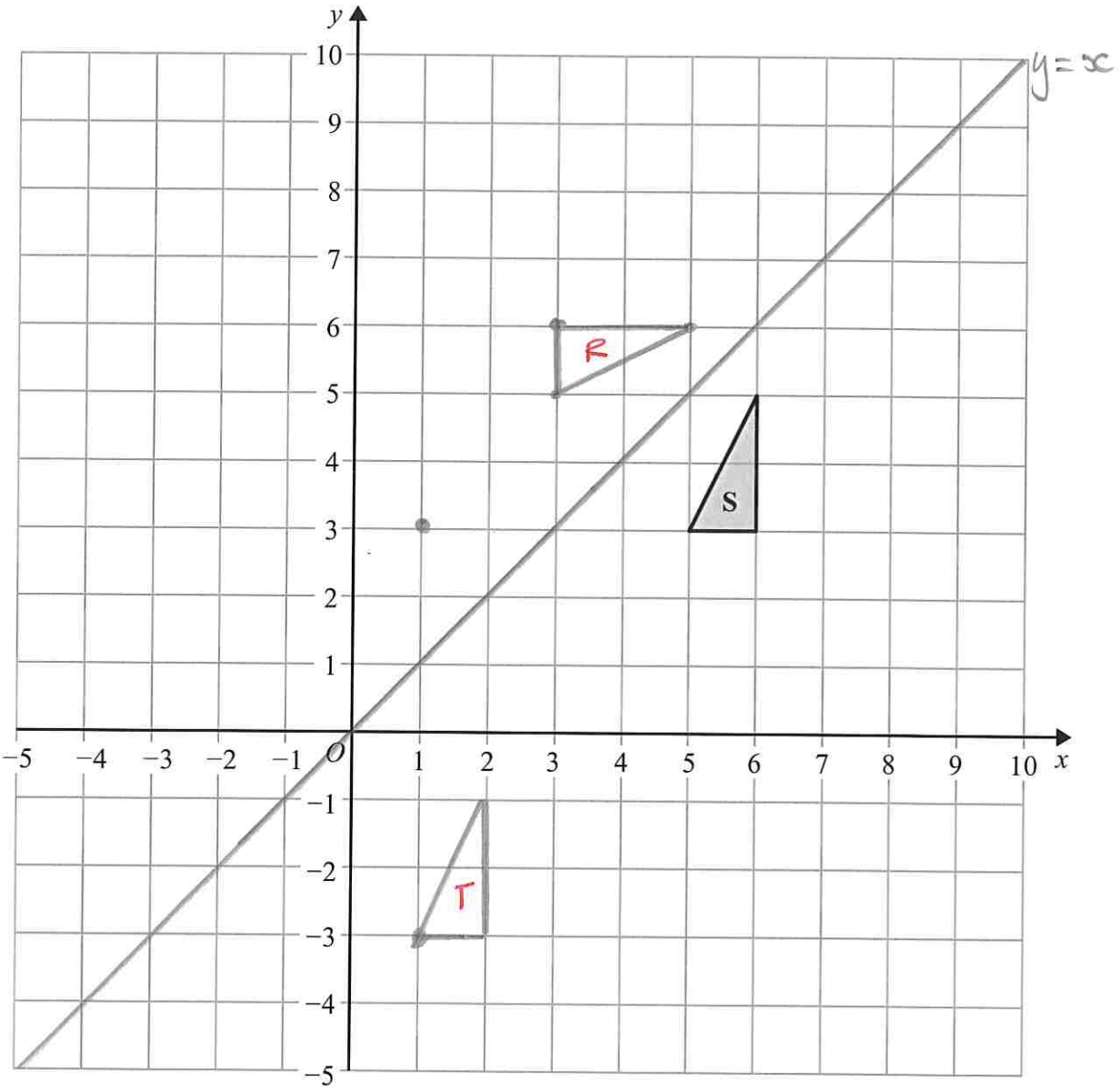
$= 460\ cm^2$

460 ..... cm^2

(Total for Question 1 is 4 marks)

Transformations

2



(*a*) Reflect triangle **S** in the line $y = x$
Label the new triangle **R**.

(2)

(*b*) Translate triangle **S** by the vector $\begin{pmatrix} -4 \\ -6 \end{pmatrix}$

Label the new triangle **T**.

(1)

(Total for Question 2 is 3 marks)

Prime Numbers

3 $E = n^2 + n + 5$

Ali thinks that the value of E will be a prime number for any whole number value of n.

Is Ali correct?

You must give a reason for your answer.

$1^2 + 1 + 5 = 7$ prime ✓

$2^2 + 2 + 5 = 11$ prime ✓

$3^2 + 3 + 5 = 17$ prime ✓

$4^2 + 4 + 5 = 25$ Not prime

No since if $n = 4$ this doesn't hold.

(Total for Question 3 is 2 marks)

Angles on parallel lines

4

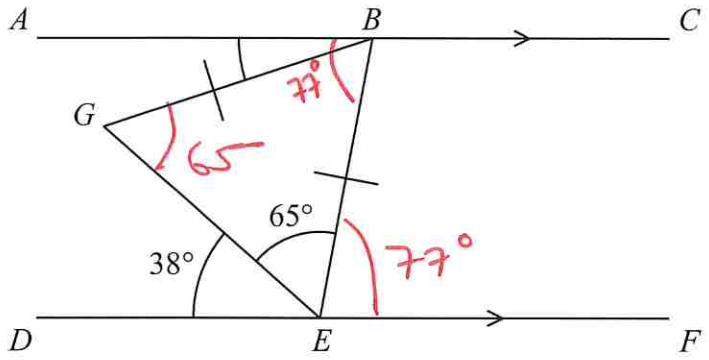


Diagram **NOT** accurately drawn

ABC and *DEF* are parallel lines.

BG = *BE*
Angle *DEG* = 38°
Angle *GEB* = 65°

Find the size of angle *ABG*.

$B\hat{E}F = 180° - 65° - 38°$
$= 77°$ — Angles on straight line = 180°

$B\hat{E}F = A\hat{B}E = 77°$ — Alternate angles equal

$B\hat{G}E = 65°$ — Isosceles triangle

$G\hat{B}E = 180° - 65° - 65°$
$= 50°$ — 180° in a triangle

$A\hat{B}G = A\hat{B}E - G\hat{B}E$
$= 77° - 50°$
$= 27°$

180 − 65 − 38 = 077°

27°

(Total for Question 4 is 3 marks)

Linear Sequences

5 Here are the first four terms of an arithmetic sequence.

2 6 10 14 18

+4 +4 +4 +4

(*a*) Find an expression, in terms of n, for the nth term of this sequence.

$= 4n+2$

$4n+2$

(2)

(*b*) Write down an expression, in terms of n, for the $(n + 1)$th term of this sequence.

Let $n = n+1$

$\therefore 4(n+1)+2$

$= 4n+4+2$

$= 4n+6$

$4n+6$

(1)

(Total for Question 5 is 3 marks)

Simplifying Algebra

6 (*a*) Simplify fully $\dfrac{20x^2y^6}{4x^2y^2} = \dfrac{20y^6}{4y^2} = \dfrac{20y^4}{4}$

$= 5y^4$

$5y^4$

(2)

(*b*) Make e the subject of the formula $h = 3e + f$

Rearranging Formula

$h = 3e+f$

$(-f)$ $h-f = 3e$

$(\div 3)$ $\dfrac{h-f}{3} = e$

$\dfrac{h-f}{3} = e$

(2)

(Total for Question 6 is 4 marks)

Standard Form

7 (*a*) Write 1 390 000 in standard form.

1 3 9 0 0 0 0.

1 . 3 9

1.39×10^{6}

.....

(1)

(*b*) Write 0.005 in standard form.

0 . 0 0 5

5

5×10^{-3}

.....

(1)

(Total for Question 7 is 2 marks)

Simultaneous Equations

8 Solve

$$3x + 2y = 15 \quad (1)$$
$$10x - 4y = 2 \quad (2)$$

Show clear algebraic working.

Same Take Opposite Plus

① × 2 :

$6x + 4y = 30$ (3)

$10x - 4y = 2$ (2) +

$16x = 32$

$(\div 16)$ $x = 2$

In ① :

$3x + 2y = 15$

$3(2) + 2y = 15$

$6 + 2y = 15$

(-6) $2y = 9$

$(\div 2)$ $y = 9/2$

$x =$ 2

$y =$ 9/2

(Total for Question 8 is 3 marks)

Circle Theorems

9

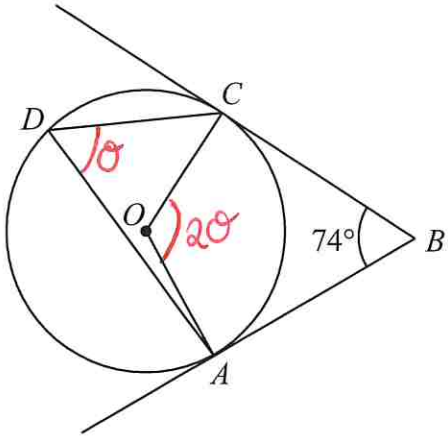


Diagram **NOT** accurately drawn

A, *C* and *D* are points on a circle, centre *O*.
AB and *CB* are tangents to the circle.

Angle *ABC* = 74°

Work out the size of angle *ADC*.
Show your working clearly.

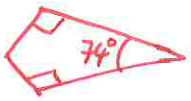
$O\hat{C}B = O\hat{A}B = 90°$

$C\hat{O}A$ is the last angle in a quadrilateral

$360° - 90° - 90° - 74° = 106°$

$106° \div 2 = 53°$

Tangent meets radius at 90°



Angles in a quadrilateral = 360°

Angle at the centre is twice that at the circumference

53 .....°

(Total for Question 9 is 3 marks)

Reverse Percentages as fractions

10 Each month Edna spends all her income on rent, on travel and on other living expenses.

She spends $\frac{1}{3}$ of her income on rent.

She spends $\frac{1}{5}$ of her income on travel.

She spends \$420 of her income on other living expenses.

Work out her income each month.

Income on living costs as a fraction (Total = $\frac{1}{1}$)

$\ldots 1 - \frac{1}{3} - \frac{1}{5}$

$1 - \frac{1}{3} - \frac{1}{5} = 1 - \frac{5}{15} - \frac{3}{15} = 1 - \frac{8}{15}$

$= \frac{7}{15}$

Cost = \$420

\$420 = $\frac{7}{15}$ of income

$(\div 7)$ \$60 = $\frac{1}{15}$ of income

$(\times 15)$ \$900 = $\frac{1}{1}$ (Total) income

\$ 900

(Total for Question 10 is 4 marks)

Index Laws

11 $128 = 4^{2x} \times 2^x$

Work out the value of x.

$4 = 2^2$

$(a^m)^n = a^{mn}$

$a^m \times a^n = a^{m+n}$

$128 = 2^7$

Drop Bases

$(\div 5)$

$128 = (2^2)^{2x} \times 2^x$

$128 = 2^{4x} \times 2^x$

$128 = 2^{5x}$

$2^7 = 2^{5x}$

$7 = 5x$

$\frac{7}{5} = x$

$x =$ $\frac{7}{5}$

(Total for Question 11 is 3 marks)

Expanding and Simplifying

12 (*a*) Simplify $(2e^2f^3)^3$

$= 2e^2f^3 \times 2e^2f^3 \times 2e^2f^3$

$8e^6f^9$

(2)

(*b*) Expand and simplify $(3x - 4y)(x + 3y)$

expand $= 3x^2 + 9xy - 4xy - 12y^2$

collect $= 3x^2 + 5xy - 12y^2$

$3x^2 + 5xy - 12y^2$

(2)

$\frac{\sqrt{a} \times a}{a^{-2}}$ can be written in the form a^k

(*c*) Find the value of k.

$\frac{\sqrt{a} \times a}{a^{-2}}$

$\sqrt{a} = a^{\frac{1}{2}}$, $a = a^1$ $= \frac{a^{\frac{1}{2}} \times a^1}{a^{-2}}$

$a^m \times a^n = a^{m+n}$ $= \frac{a^{\frac{3}{2}}}{a^{-2}}$

$\frac{a^m}{a^n} = a^{m-n}$ $= a^{\frac{7}{2}}$

$k = \frac{7}{2}$

(2)

(*d*) Simplify $\frac{2^n - 1}{4^n - 1}$

$4^n - 1 = (2^n - 1)(2^n + 1)$ $= \frac{2^n - 1}{(2^{n\text{-}n} - 1)(2^n + 1)}$

$= \frac{1}{2^n + 1}$

$\frac{1}{2^n + 1}$

(2)

(Total for Question 12 is 8 marks)

Independent Probability Trees

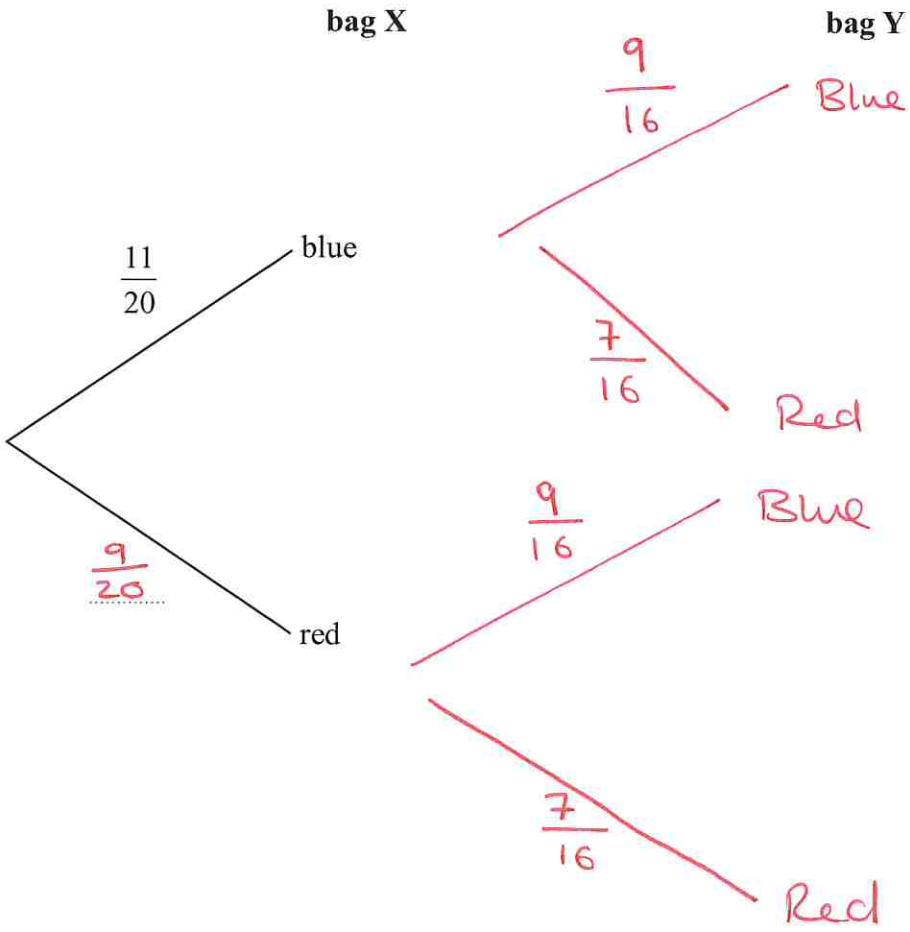
13 There are two bags of counters, bag **X** and bag **Y**.

There are 20 counters in bag **X**.
11 of the counters are blue and the rest are red.

There are 16 counters in bag **Y**.
9 of the counters are blue and the rest are red.

Arkady takes at random a counter from bag **X** and takes at random a counter from bag **Y**.

(*a*) Complete the probability tree diagram.



(3)

(*b*) Work out the probability that the two counters are both red.

$P(R,R) = \frac{9}{20} \times \frac{7}{16}$

$= \frac{63}{320}$

$\frac{63}{320}$

.....

(2)

(*c*) Work out the probability that the two counters are both red or are both blue.

$P(\text{same colour}) = P(R,R) + P(B,B)$

$= \frac{63}{320} + \frac{11}{20} \times \frac{9}{16}$

$= \frac{63}{320} + \frac{99}{320}$

$= \frac{162}{320}$

$\frac{162}{320}$

.....

(3)

(Total for Question 13 is 8 marks)

Histograms

14 The table gives information about the areas, in hectares, of some farms in Spain.

Area (A hectares)		Frequency	f.d
$0 < A \leq 20$	20	40	2
$20 < A \leq 50$	30	90	3
$50 < A \leq 100$	50	140	2.8
$100 < A \leq 300$	200	140	0.7
$300 < A \leq 350$	50	40	0.8

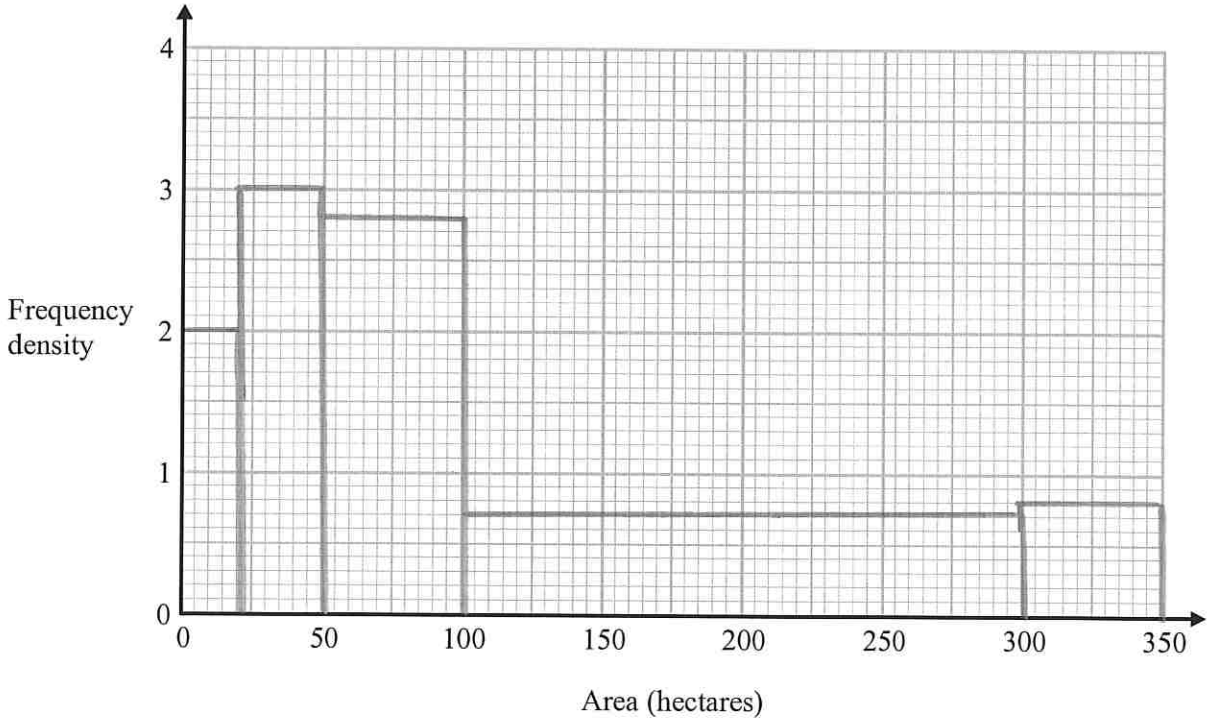
f.d = f / cw

$\frac{140}{50} = \frac{14}{5} = \frac{28}{10} = 2.8$

$\frac{140}{200} = \frac{14}{20} = \frac{7}{10} = 0.7$

$\frac{40}{50} = \frac{4}{5} = 0.8$

On the grid, draw a histogram for this information.



(Total for Question 14 is 3 marks)

Recurring Decimals

15 (*a*) Use algebra to show that $0.4\dot{3}\dot{6} = \frac{24}{55}$

Let $x = 0.436363636\ldots$

$10x = 4.363636363\ldots$

$100x = 43.636363636\ldots$

$1000x = 436.363636363\ldots$

$1000x - 10x = 436.3636363\ldots$ (−)

$4.3636363\ldots$

$990x = 432$

$(\div 990)$ $x = \frac{432}{990} = \frac{216}{495} = \frac{72}{165} = \frac{24}{55}$ □. **(2)**

(*b*) Show that $\frac{\sqrt{20}+\sqrt{80}}{\sqrt{3}}$ can be expressed in the form $\sqrt{a}$ where a is an integer.

Show your working clearly.

Simplifying Surds (Rationalising)

$\sqrt{20} = \sqrt{4}\sqrt{5}$

$\sqrt{80} = \sqrt{16}\sqrt{5}$

$= \frac{2\sqrt{5} + 4\sqrt{5}}{\sqrt{3}}$

$= \frac{6\sqrt{5}}{\sqrt{3}}$

Rationalise $\left(\times\frac{\sqrt{3}}{\sqrt{3}}\right)$ $= \frac{6\sqrt{5}}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}}$

$\sqrt{a}\sqrt{b} = \sqrt{a \times b}$ $= \frac{6\sqrt{15}}{3}$

$= 2\sqrt{15}$

$2 = \sqrt{4}$ $= \sqrt{4}\sqrt{15}$

$\sqrt{a}\sqrt{b} = \sqrt{a \times b}$ $= \sqrt{60}$

(3)

(Total for Question 15 is 5 marks)

16 Two functions, f and g are defined as

$$f : x \mapsto 1 + \frac{1}{x} \qquad \text{for } x > 0$$

$$g : x \mapsto \frac{x+1}{2} \qquad \text{for } x > 0$$

Given that h = fg

express the inverse function h^{-1} in the form $h^{-1} : x \mapsto \ldots$

$h(x) = f(g(x))$

$f(g(x)) = 1 + \frac{1}{\frac{x+1}{2}}$

$= 1 + \frac{2}{x+1} = h(x)$

WANT: $h^{-1}(x)$

$y = 1 + \frac{2}{x+1}$

(-1) $y - 1 = \frac{2}{x+1}$

$(\times (x+1))$ $(x+1)(y-1) = 2$

$(\div (y-1))$ $x + 1 = \frac{2}{y-1}$

(-1) $x = \frac{2}{y-1} - 1$

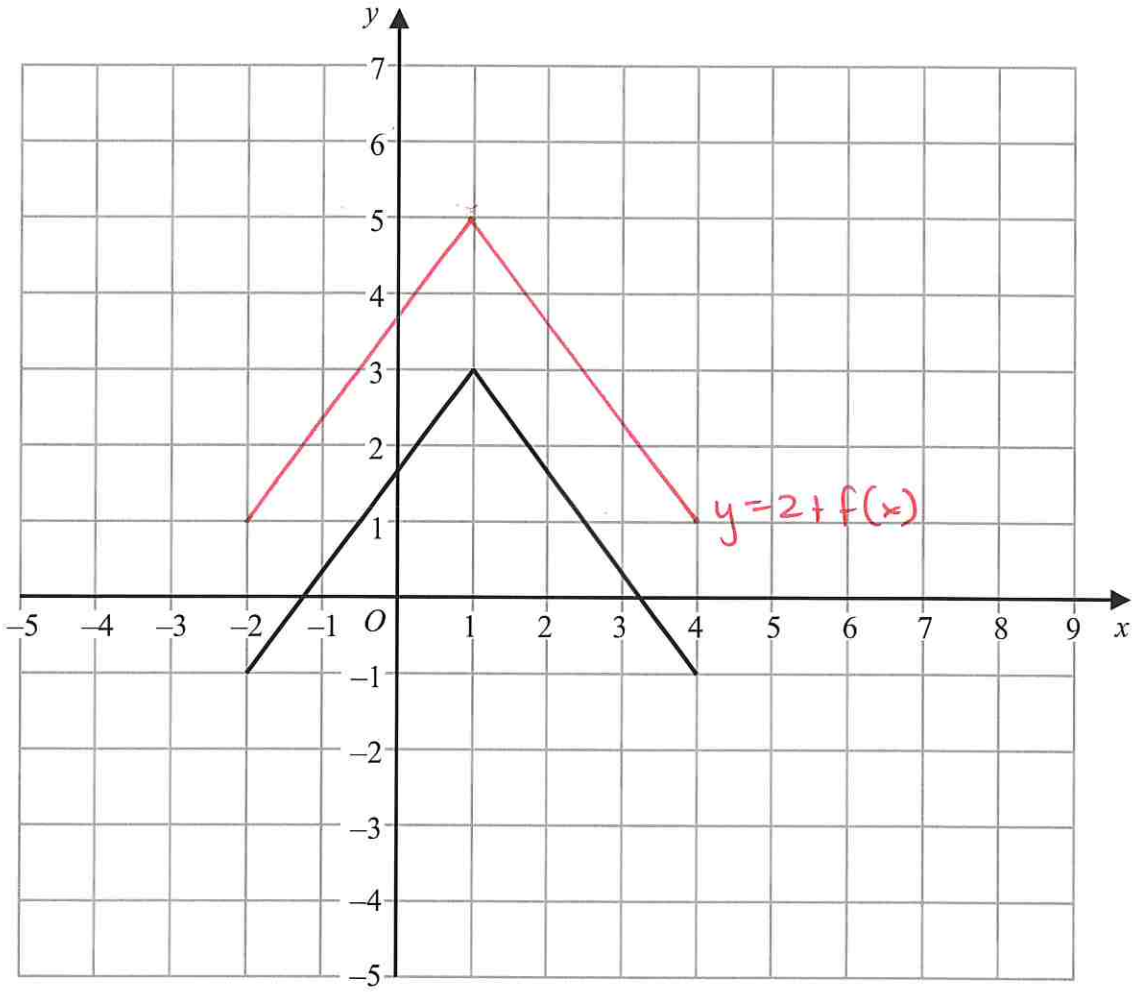
Notation $h^{-1}(x) = \frac{2}{x-1} - 1$

$h^{-1} : x \mapsto \frac{2}{x-1} - 1$

(Total for Question 16 is 4 marks)

Transformations of Graphs

17 Here is the graph of $y = f(x)$

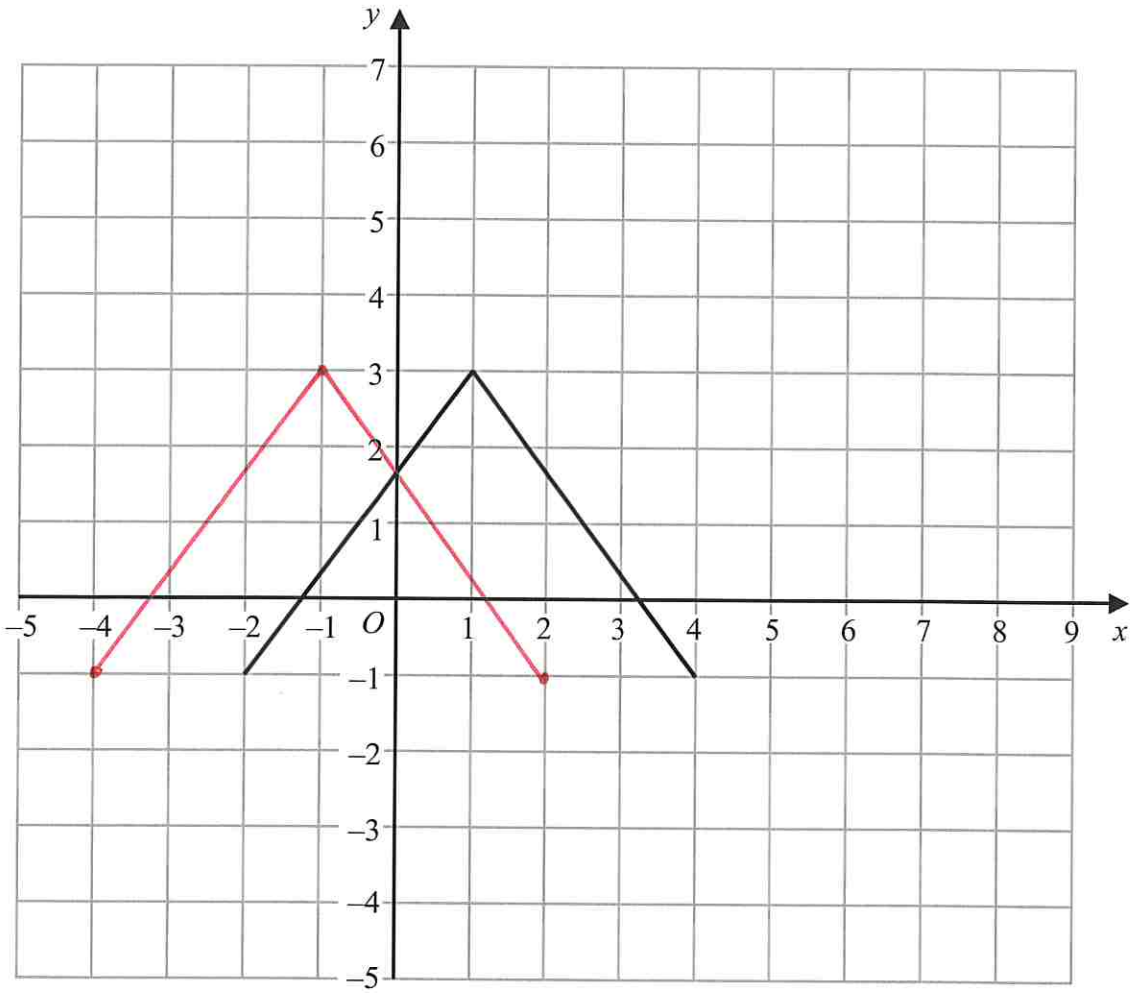


(*a*) On the grid above, draw the graph of $y = 2 + f(x)$

(2)

f(x)+a... Translation of vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$

Here is the graph of $y = f(x)$



(*b*) On the grid above, draw the graph of $y = f(-x)$

Inputs are negated... Reflection over y-axis.

(2)

(Total for Question 17 is 4 marks)

Expanding Triple Brackets

18 (*a*) Show that $x(x-1)(x+1) = x^3 - x$

$(x-1)(x+1) = x^2 + x - x - 1 = x^2 - 1$

$x(x^2-1) = x^3 - x$ □

(1)

(*b*) Prove that the difference between a whole number and the cube of this number is always a multiple of 6.

Algebraic Proof

Let a number (integer) be $= x$

Cube $= x^3$

Difference $= x^3 - x$

from part (a)

$= x(x-1)(x+1)$

If x = even:

x is a ~~multiple~~ factor of 2

Then $x+1$ or $x-1$ is a multiple of 3

e.g. 15 16 17
19 20 21
17 (18) 19

OR x is a multiple of both 2 and 3.

∴ $x(x-1)(x+1)$ is always a multiple of $2 \times 3 = 6$. □

(3)

(Total for Question 18 is 4 marks)

[This question wouldn't appear on a GCSE (9-1) paper but it's been left in as a challenge problem to solve!]

19 Work out the sum of the multiples of 3 between 1 and 1000.

Linear Sequence

(1) (2) (3) (4) ... ... (331) (332) (333)

3 6 9 12 ... ... 993 996 999

→ +3 → +3 → +3 → +3 → +3

Sum of linear sequence $= \frac{n}{2}(\text{first term} + \text{last term})$

$\text{Sum} = \frac{333}{2}(3+999)$

$= 166833$

* WHY DOES THIS WORK?

Average term in the sequence is the term directly half way between the first and last term. There are 333 terms ($= 166.5^{\text{th}}$ term)

Each combination of highest + lowest terms = 1002

e.g: 3 + 999 = 1002

6 + 996 = 1002

There are 166.5 pairs like this, all equal to 1002

$166.5 \times 1002 = 166833$

166833

(Total for Question 19 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS