MODEL SOLUTIONS

RE4

Q23.

GCSE Mathematics

Practice Tests: Set 8

Paper 3H (Calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Averages and Range

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Three numbers have

a mean of 17
a median of 20
a range of 27.

Find the three numbers.

Suppose the numbers are x y z

mean = 17 | $\frac{x+y+z}{3} = 17$ $\therefore$ $x+y+z = 51$

median = 20 | $y = 20 \Rightarrow \therefore$ $x+20+z = 51$

$x+z = 31$

range = 27 | $\therefore x = 2, z = 29$

2 , 20 , 29

(Total for Question 1 is 3 marks)

Angles in Parallel Lines

2

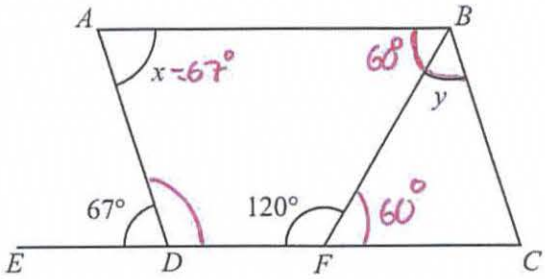


Diagram **NOT** accurately drawn

ABCD is a parallelogram.
EDFC is a straight line.

(*a*) (i) Write down the size of angle *x*.

67 .....°

(ii) Give a reason for your answer.

Alternate angles on parallel lines equal.

(2)

(*b*) Work out the size of angle *y*.

ABCD is a Parallelogram

Alternate angles

$A\hat{B}C + B\hat{A}D = 180°$ (co-interior)

$C\hat{F}B = 180° - 120° = 60° = A\hat{B}F$

$67° + 60° + y° = 180°$

$127 + y° = 180°$

$y° = 53°$

53 .....°

(2)

(Total for Question 2 is 4 marks)

Frequency Tables

3 The table gives information about the number of trees in each of 20 gardens.

Number of trees	C.F	Frequency
0	2	2
1	9	7
2	12	3
3	16	4
4	19	3
5	20	1

Total Trees
0
7
6
12
12
5
TOTAL: 42

(*a*) Work out the total number of trees in these gardens.

42

(2)

(*b*) Find the median number of trees in these gardens.

median = 10th term

00111111122

median = 2

2

(1)

(Total for Question 3 is 3 marks)

---

4 Charlotte earns £8.50 per hour.
She gets a pay rise of 6%

Percentage Increase

Work out how much Charlotte earns per hour after her pay rise.

100% = £8.50
(÷100) 1% = £0.085
(×6) 6% = £0.51
£8.50 + £0.51 = £9.01

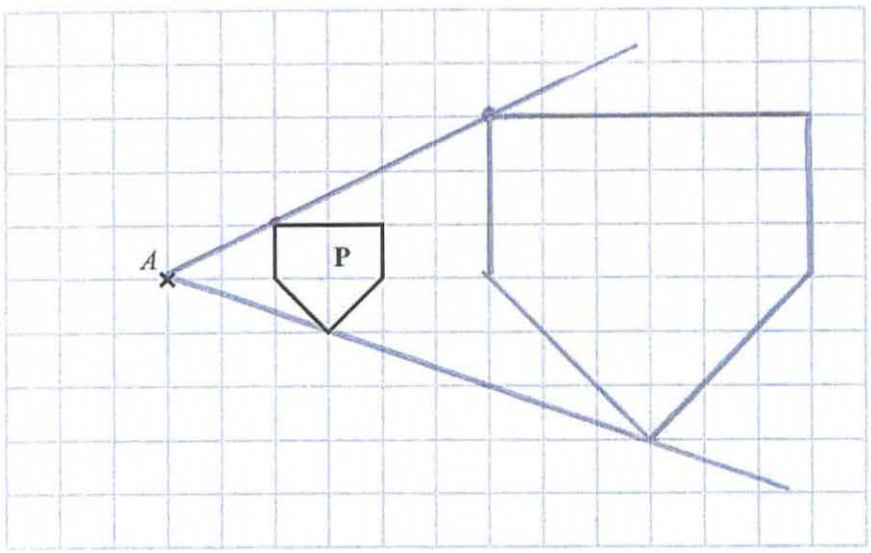
£.....

(Total for Question 4 is 3 marks)

Transformations: Enlargements

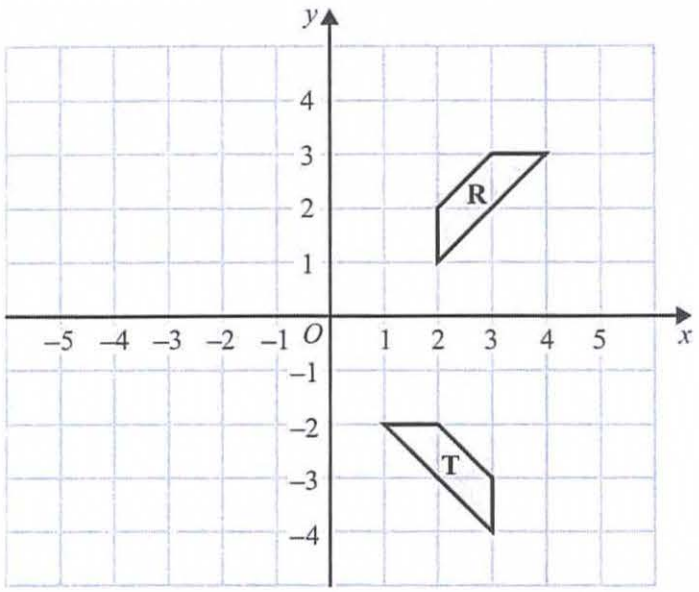
$\begin{pmatrix}2\\1\end{pmatrix}$ SF 3 $3\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}6\\3\end{pmatrix}$

5



(*a*) On the grid, enlarge shape **P** with scale factor 3 and centre *A*.

(2)



(*b*) Describe fully the single transformation that maps shape **R** onto shape **T**.

Rotation 90° clockwise centre (0, 0)

(3)

(Total for Question 5 is 5 marks)

Speed Distance Time

6 A plane flew from Sydney to Wellington.

The distance the plane flew was 2240 km.
The average speed of the plane was 805 km/h.

Work out the time taken by the plane to fly from Sydney to Wellington.
Give your answer in hours and minutes, correct to the nearest minute.

$D = 2240$ km
$S = 805$ km/h
$T = ?$

$T = \frac{D}{S}$

$= \frac{2240\text{km}}{805\text{km/h}}$

$= 2.7826\ldots$ hours

$= 47$ minutes...

$\therefore$ 2 hours 47 mins

0.7826... of an hour (×60)

..... hours ..... minutes

(Total for Question 6 is 3 marks)

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7 Solve the simultaneous equations

Simultaneous Equation

$$y = 4x \quad (1)$$
$$7x - y = -13.5 \quad (2)$$

Show clear algebraic working.

Sub (1) into (2)

(2): $7x - y = -13.5$

$7x - (4x) = -13.5$

$3x = -13.5$

$(\div 3)$ $x = -4.5$

Put $x = -4.5$ into (1):

$y = 4(-4.5)$

$y = -18$

$x =$ −4.5

$y =$ −18

(Total for Question 7 is 3 marks)

SOHCAHTOA Bearings

8 A, B and C are three cities.

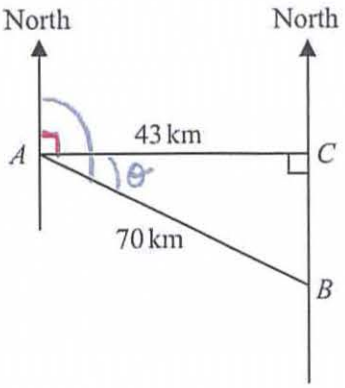
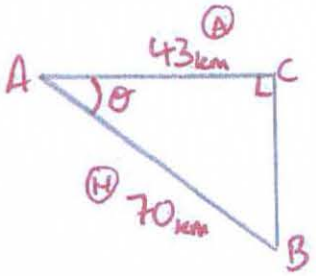


Diagram **NOT** accurately drawn

City C is due east of city A and due north of city B.
City A is 43 km from city C and 70 km from city B.

Work out the bearing of city B from city A.
Give your answer correct to the nearest degree.



SOHCAHTOA

$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{43}{70}$

$\theta = 52.0996...$

$\theta = 52$ (nearest degree)

Shift cos

Bearing of B from A

$= 90 + \theta = 142°$

142 °

(Total for Question 8 is 4 marks)

Simplifying

9 Simplify $(3a^2b^4)^3$

$= 3a^2b^4 \times 3a^2b^4 \times 3a^2b^4$

$= 27a^6b^{12}$

$27a^6b^{12}$

(Total for Question 9 is 2 marks)

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10 Solve the inequalities $-5 \leq 2p + 3 < 13$

Solving Inequalities

$-5 \leq 2p+3 < 13$

(-3) $\quad -8 \leq 2p < 10$

$(\div 2)$ $\quad -4 \leq p < 5$

$-4 \leq p < 5$

(Total for Question 10 is 3 marks)

Interquartile Range (Raw Data)

11 A group of 15 businessmen were asked to give the number of different countries they had each visited on business.

Here are the results.

0 1 3 3 4 6 7 8 8 9 10 10 11 12 14

Work out the interquartile range of the number of countries visited.

$UQ = \frac{3(n+1)}{4} = 12^{th}$ term $= 10$

$LQ = \frac{(n+1)}{4} = 4^{th}$ term $= 3$

$IQR = UQ - LQ$

$= 10 - 3$

$= 7$

7

(Total for Question 11 is 2 marks)

---

12 Solve $\frac{5x-2}{3} + \frac{3-5x}{4} = 2$

Solving Algebraic Fractions Equation

Show clear algebraic working.

$\frac{5x-2}{3} + \frac{3-5x}{4} = 2$

Simplify (cross-multiply): $\frac{4(5x-2)+3(3-5x)}{12} = 2$

(×12): $4(5x-2)+3(3-5x) = 24$

expand: $20x - 8 + 9 - 15x = 24$

collect: $5x + 1 = 24$

(−1): $5x = 23$

(÷5): $x = 4.6$

$x =$ 4.6

(Total for Question 12 is 4 marks)

Standard Form Calculations

13 The population of China is 1.4×10^9
The population of Morocco is 3.5×10^7

The population of China is k times the population of Morocco.

Work out the value of k.

China / Morocco

$\frac{1.4 \times 10^9}{3.5 \times 10^7} = 40$

USE YOUR CALCULATOR

40

(Total for Question 13 is 2 marks)

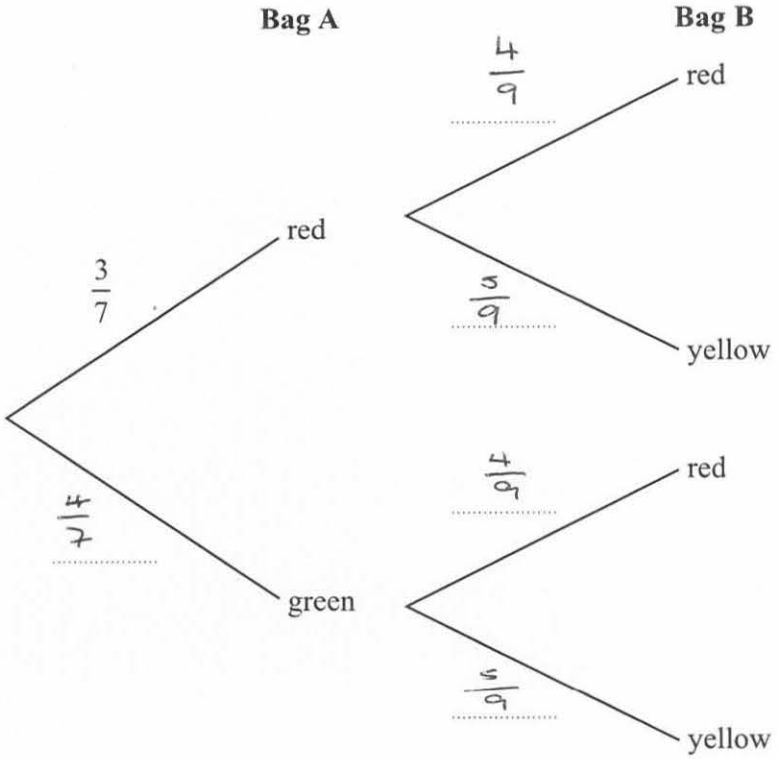
Independent Probability Trees

14 Genevieve has two bags of marbles, bag **A** and bag **B**.

In bag **A** there are only 3 red marbles and 4 green marbles.
In bag **B** there are only 4 red marbles and 5 yellow marbles.

Genevieve takes at random one marble from each bag.

(*a*) Complete the probability tree diagram.



(2)

(*b*) Work out the probability that Genevieve takes two red marbles.

$P(R,R) = \frac{3}{7} \times \frac{4}{9}$

$= \frac{12}{63}$

$\frac{12}{63}$

(2)

(Total for Question 14 is 4 marks)

Rearranging Complex Formula

15 Make w the subject of the formula $p = \sqrt{\dfrac{w+4}{w-2}}$

	$p = \sqrt{\frac{w+4}{w-2}}$
$(\text{ANS})^2$	$p^2 = \frac{w+4}{w-2}$
$(\times(w-2))$	$p^2(w-2) = w+4$
expand	$p^2w - 2p^2 = w+4$
$(-w)$	$p^2w - 2p^2 - w = 4$
$(+2p^2)$	$p^2w - w = 4+2p^2$
factorise	$w(p^2-1) = 4+2p^2$
$(\div(p^2-1))$	$w = \frac{4+2p^2}{p^2-1}$

$w = \dfrac{4+2p^2}{p^2-1}$

.....

(Total for Question 15 is 4 marks)

Histograms

16 The incomplete histogram and table give information about the lengths of time, in minutes, that some people spent at an airport.

$fd = \frac{f}{c \cdot w}$ $f = f \cdot d \times c \cdot w$

Time (t minutes)		Frequency	f·d
$0 < t \leq 50$	50	10	0.2
$50 < t \leq 80$	30	15	0.5
$80 < t \leq 120$	40	28	0.7
$120 < t \leq 180$	60	18	0.3
$180 < t \leq 240$	60	12	0.2
$240 < t \leq 320$	80	8	0.1

$10 \div 50 = 0.2$

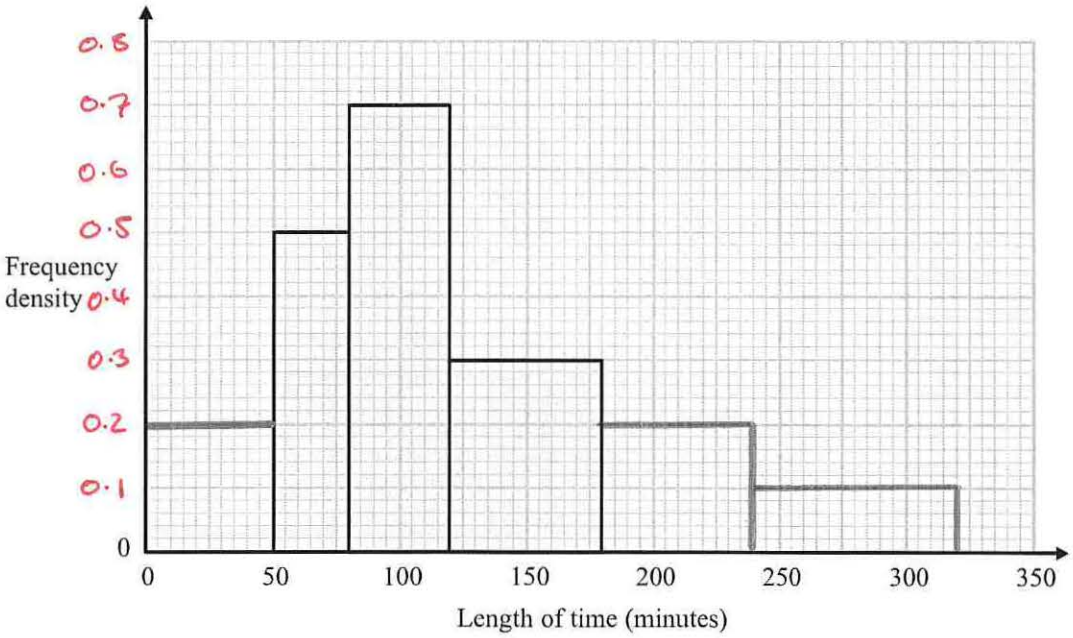
$15 \div 30 = 0.5$

$40 \times 0.7 = 28$

$60 \times 0.3 = 18$

$12 \div 60 = 0.2$

$8 \div 80 = 0.1$



(*a*) Use the histogram to complete the table.

(2)

(*b*) Use the table to complete the histogram.

(2)

(Total for Question 16 is 4 marks)

17

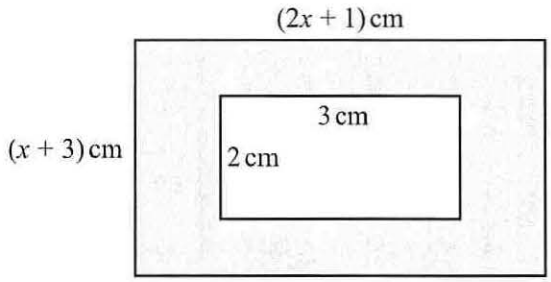


Diagram **NOT** accurately drawn

The diagram shows a rectangular piece of card with length $(2x + 1)$ cm and width $(x + 3)$ cm.
A rectangle of length 3 cm and width 2 cm is cut out of the card.

The area of card that remains, shown shaded in the diagram, is 45 cm^2

(*a*) Show that $2x^2 + 7x - 48 = 0$

Area large | $(2x+1)(x+3) = 2x^2+6x+x+3$
$= 2x^2 + 7x + 3$

Area small | $3 \times 2 = 6$

Shaded area | $A = 2x^2+7x+3-6$

$A = 45$ | $45 = 2x^2+7x-3$

(45) | $= 2x^2+7x-48$ □

(2)

(*b*) Find the value of x.
Show your working clearly.
Give your value of x correct to 3 significant figures.

Quadratic Formula

$a = 2$
$b = 7$
$c = -48$
Quadratic formula | $2x^2+7x-48=0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-(7) \pm \sqrt{(7)^2-4(2)(-48)}}{2(2)}$

$x = 3.45$

$x_+ = 3.45$ (3.s.f)

$x_- = -6.95$ (3.s.f)

.....

(3)

(Total for Question 17 is 5 marks)

Conclusion | Dimensions MUST be positive $\therefore x = 3.45$

Recurring Decimals

18 Use algebra to show that the recurring decimal $0.2\dot{7}\dot{8} = \frac{46}{165}$

let $x = 0.2\dot{7}\dot{8}$

$= 0.278787878\ldots$

$10x = 2.787878787\ldots$

$100x = 27.878787878\ldots$

$1000x = 278.787878787\ldots$

$1000x - 10x = 278.787878\ldots - 2.787878\ldots$ (−)

$990x = 276$

$990x = 276$

$x = \frac{276}{990} = \frac{138}{495} = \frac{46}{165}$

(Total for Question 18 is 2 marks)

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19 Express $\frac{x+3}{x-4} - \frac{x+4}{x-3}$ as a single fraction.

Algebraic Fractions

Simplify your answer.

cross multiply: $\frac{(x+3)(x-3) - (x+4)(x-4)}{(x-4)(x-3)}$

expand top: $= \frac{x^2 - 3x + 3x - 9 - [x^2 - 4x + 4x - 16]}{(x-4)(x-3)}$

collect/expand: $= \frac{x^2 - 9 - [x^2 - 16]}{(x-4)(x-3)}$

$= \frac{x^2 - 9 - x^2 + 16}{(x-4)(x-3)}$

$= \frac{7}{(x-4)(x-3)}$

$\frac{7}{(x-4)(x-3)}$

(Total for Question 19 is 3 marks)

Index Laws / Quadratics

20 $\frac{5^{n^2}}{5^6} \times \frac{5^{n^2-5n}}{5^3} = 125$ where $n > 0$

Work out the value of n.
Show clear algebraic working.

$a^m \times a^n = a^{m+n}$

$= \frac{5^{n^2+n^2-5n}}{5^{6+3}} = 125$

$= \frac{5^{2n^2-5n}}{5^9} = 125$

$a^m \div a^n = a^{m-n}$

$= 5^{2n^2-5n-9} = 125$

$125 = 5^3$

$5^{2n^2-5n-9} = 5^3$

Drop Bases

$\therefore 2n^2 - 5n - 9 = 3$

(-3)

$2n^2 - 5n - 12 = 0$

factorise

$(2n+3)(n-4) = 0$

solve

$2n + 3 = 0$ OR $n - 4 = 0$

$2n = -3$ $\quad n = 4$

$n = -3/2$

Given $n > 0$

$\therefore n = 4$

$n =$ 4

(Total for Question 20 is 5 marks)

21

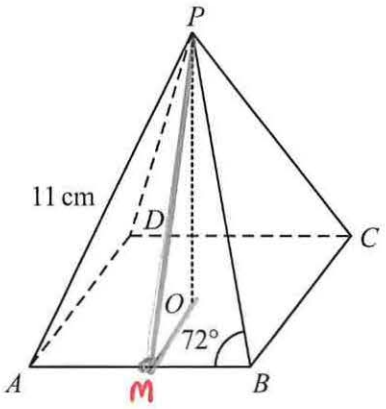
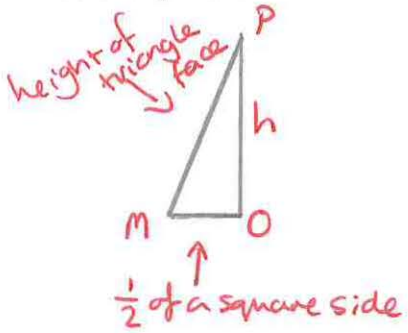


Diagram **NOT** accurately drawn

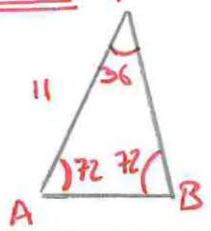


The diagram shows a pyramid with a horizontal square base.
The vertex, P, of the pyramid is vertically above the centre, O, of the base.
The triangular faces of the pyramid are congruent isosceles triangles.

In triangle ABP
$PA = PB = 11$ cm and angle $PBA = 72°$

Work out the height, OP, of the pyramid.
Give your answer correct to 3 significant figures.

SQUARE SIDE



Sine Rule

$\hat{BAP} = \hat{PBA} = 72°$ (isosceles triangle)

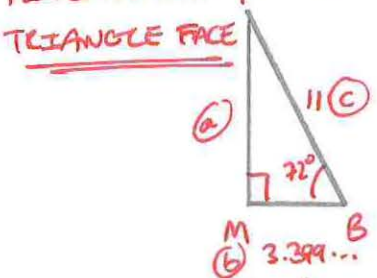
$\hat{APB} = 180° - 72° - 72°$ (180° in a triangle)
$= 36°$

$\frac{AB}{\sin 36} = \frac{11}{\sin 72}$

$AB = \frac{11}{\sin 72} \times \sin 36 = 6.7983...$

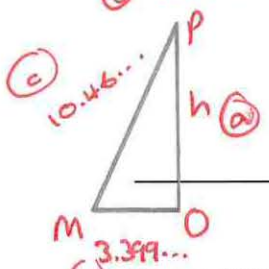
$BM = \frac{1}{2}AB = 3.399...$

HEIGHT OF TRIANGLE FACE (×sin(36))



$a^2 + b^2 = c^2$
$a^2 = c^2 - b^2$
$a^2 = 11^2 - 3.399^2 = 109.44...$ $\therefore a = \sqrt{ANS} = 10.46...$

Pythagoras again... $a^2 + b^2 = c^2$
$a^2 = c^2 - b^2$



$a^2 = 109.44... - 11.55...$

$a^2 = 97.89...$

$\sqrt{ANS}$ $a = 9.89$ cm

.....cm

(Total for Question 21 is 4 marks)

Angles in Polygons and Arc length

22

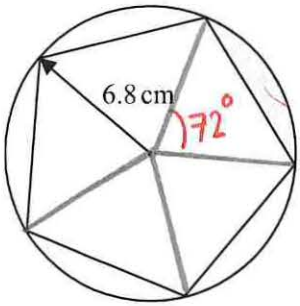


Diagram **NOT** accurately drawn

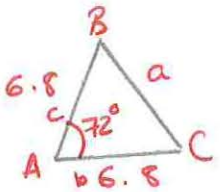
A regular pentagon is drawn inside a circle of radius 6.8 cm.
Each vertex of the pentagon lies on the circle.

Find the perimeter of the region shown shaded in the diagram.
Give your answer correct to 3 significant figures.

Angles around a point = 360°

Regular Pentagon ... 5 triangles.

$360 \div 5 = 72°$



cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6.8^2 + 6.8^2 - 2(6.8)(6.8)\cos(72)$

$a^2 = 63.902...$

$a = 7.993..$

√ANS

Arc length $= \frac{\theta}{360} \times \pi d$ $(d = 6.8 \times 2)$

Arc length $= \frac{72}{360} \times \pi \times 13.6$

$= 8.545...$

Total perimeter

$7.993... + 8.545... = 16.5$ (3s.f)

16.5 .....cm

(Total for Question 22 is 4 marks)

Bounds

23 A box is in the shape of a cube of side 11.5 cm, correct to 1 decimal place.
A solid spherical ball has radius 5.1 cm, correct to the nearest millimetre.
The ball is placed inside the box and the box is closed.

Work out the upper bound for the volume of the box that is **not** occupied by the ball.

Show your working clearly.
Give your answer correct to the nearest whole number.

Length error

$11.5 \overset{+0.05}{\nearrow} 11.55$ max
$\underset{-0.05}{\searrow} 11.45$ min

$0.1 \div 2 = 0.05$

Radius error

$5.1 \overset{+0.05}{\nearrow} 5.15$ max
$\underset{-0.05}{\searrow} 5.05$ min

$0.1 \div 2 = 0.05$

LOGIC:

We want as much space empty as possible in the box, so we want the ball to be as small as possible and we want the box to be as big as possible.

(radius_{min})

(length_{max})

$\text{ball}_{min} \left(V = \frac{4\pi r^3}{3}\right)$

$\text{ball}_{min} = \frac{4\pi(5.05)^3}{3} = 539.46\ldots \text{cm}^3$

$\text{box}_{max} (V = l \times l \times l)$

$\text{box}_{max} = 11.55 \times 11.55 \times 11.55 = 1540.79\ldots \text{cm}^3$

Empty space max

$= 1540.79\ldots \text{cm}^3 - 539.46\ldots \text{cm}^3$

$= 1001.33\ldots \text{cm}^3$

$= 1001 \text{cm}^3$

1001 ..... cm^3

(Total for Question 23 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS