MODEL SOLUTIONS

REV

Q15.

GCSE Mathematics

Practice Tests: Set 8

Paper 1H (Non-calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser. Tracing paper may be used.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators must not be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Standard Form

Answer all TWENTY questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (*a*) Write 8×10^4 as an ordinary number.

80000

.....

(1)

(*b*) Work out $(3.5 \times 10^5) \div (7 \times 10^8)$
Give your answer in standard form.

split

$3.5 \div 7 = 0.5$

$10^5 \div 10^8 = 10^{-3}$

Combine

$= 0.5 \times 10^{-3}$

Standard form

$= 5 \times 10^{-1} \times 10^{-3}$

$= 5 \times 10^{-4}$

5×10^{-4}

.....

(2)

(Total for Question 1 is 3 marks)

Index Laws

2 (*a*) Simplify $y^5 \times y^9$

$a^m \times a^n = a^{m+n}$

y^{14}

.....

(1)

(*b*) Simplify $(2m^3)^4$

$= 2m^3 \times 2m^3 \times 2m^3 \times 2m^3$

$= 16m^{12}$

$16m^{12}$

.....

(2)

(*c*) Solve $5(x + 3) = 3x - 4$
Show clear algebraic working.

Solving Equations

	$5(x+3) = 3x-4$
Expand	$5x+15 = 3x-4$
$(-3x)$	$2x+15 = -4$
(-15)	$2x = -19$
$(\div 2)$	$x = -19/2$

$x =$ $-19/2$

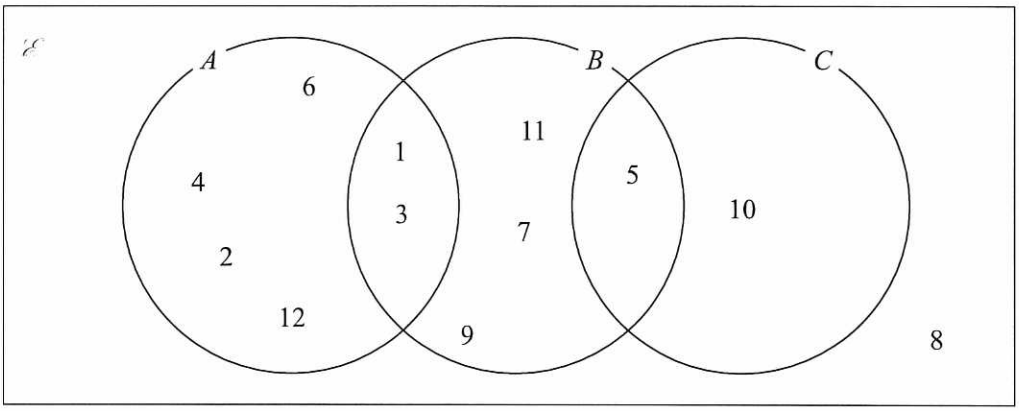
.....

(3)

(Total for Question 2 is 6 marks)

Venn Diagrams (Set Theory)

3 Here is a Venn diagram.



Write down the numbers that are in the set

(i) A

1, 2, 3, 4, 6, 12

(ii) $B \cup C$

OR

1, 3, 5, 7, 9, 10, 11

(2)

(Total for Question 3 is 2 marks)

Rearranging Formula

4 (*a*) Make *a* the subject of the formula $M = ac - bd$

$M = ac - bd$

$(+bd)$ $M + bd = ac$

$(\div c)$ $\frac{M + bd}{c} = a$

$\frac{M + bd}{c} = a$

(2)

(*b*) Solve the inequality $5x - 4 < 39$

Solving inequalities

$5x - 4 < 39$

$(+4)$ $5x < 43$

$(\div 5)$ $x < \frac{43}{5}$

$x < \frac{43}{5}$

(2)

(*c*) Factorise fully $18e^2f^3 - 12e^3f$

Factorising

6 is a factor

e^2 is a factor

f is a factor

$\therefore = 6e^2f(3f^2 - 2e)$

$6e^2f(3f^2 - 2e)$

(2)

(Total for Question 4 is 6 marks)

Factorising / Solving Quadratics

5 (*a*) Factorise $x^2 + 2x - 24$

Product = -24
Sum = 2

$= (x+6)(x-4)$

$(x+6)(x-4)$

(2)

(*b*) Hence, solve $x^2 + 2x - 24 = 0$

from (a)

$(x+6)(x-4) = 0$

Solve

$x+6 = 0$ or $x-4 = 0$

$x = -6$ or $x = 4$

$x = -6$ or 4

(1)

(Total for Question 5 is 3 marks)

Circle Theorems

6

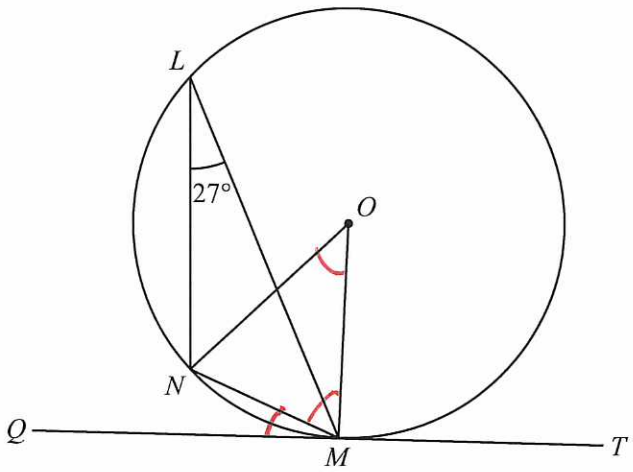


Diagram **NOT** accurately drawn

L, *M* and *N* are points on a circle, centre *O*.
QMT is the tangent to the circle at *M*.

(*a*) (i) Find the size of angle *NOM*.

2x27 = 54

54 .....°

(ii) Give a reason for your answer.

Angle at the centre is twice that at the circumference.

(2)

(*b*) (i) Find the size of angle *NMQ*.

27° .....

(ii) Give a reason for your answer.

Alternate segment theorem.

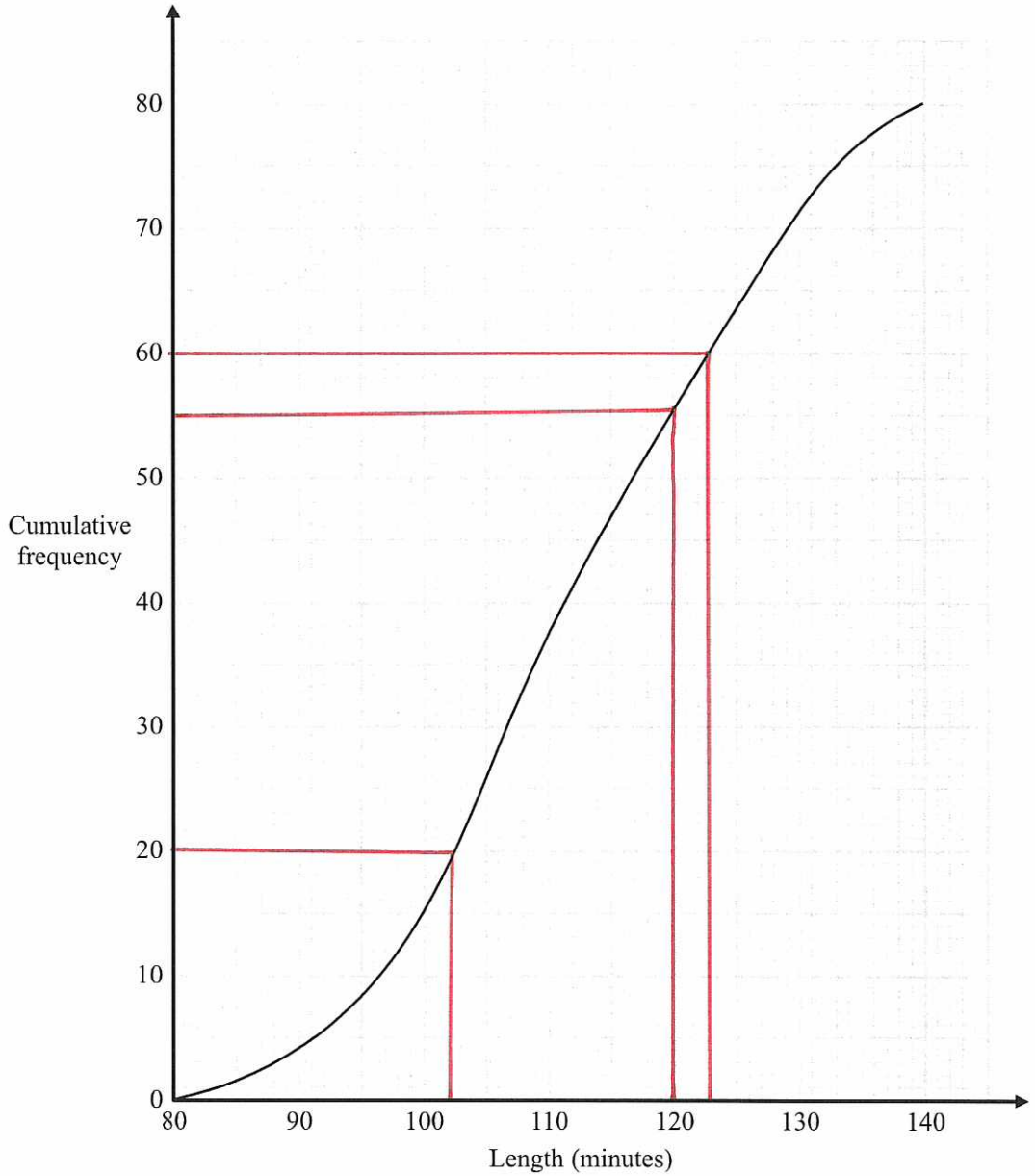
(2)

Always look for a split 90° angle made at the tangent!

(Total for Question 6 is 4 marks)

Cumulative Frequency

7 The cumulative frequency graph shows information about the length, in minutes, of each of 80 films.



(*a*) Use the graph to find an estimate for the interquartile range.

IQR = UQ − LQ = 123 − 102 = 21

UQ = $\frac{3n}{4}$ = 60th term = 123

LQ = $\frac{n}{4}$ = 20th term = 102

21 ..... minutes

(2)

Clare says,

"More than 35% of these films are over 120 minutes long."

(*b*) Is Clare correct?
Give a reason for your answer.

55 films are up until 120 minutes long

80 − 55 = 25 ∴ 25 films are over.

$\frac{25}{80}$ (×100) = 31.25%

.....

∴ Clare is incorrect.

.....

(3)

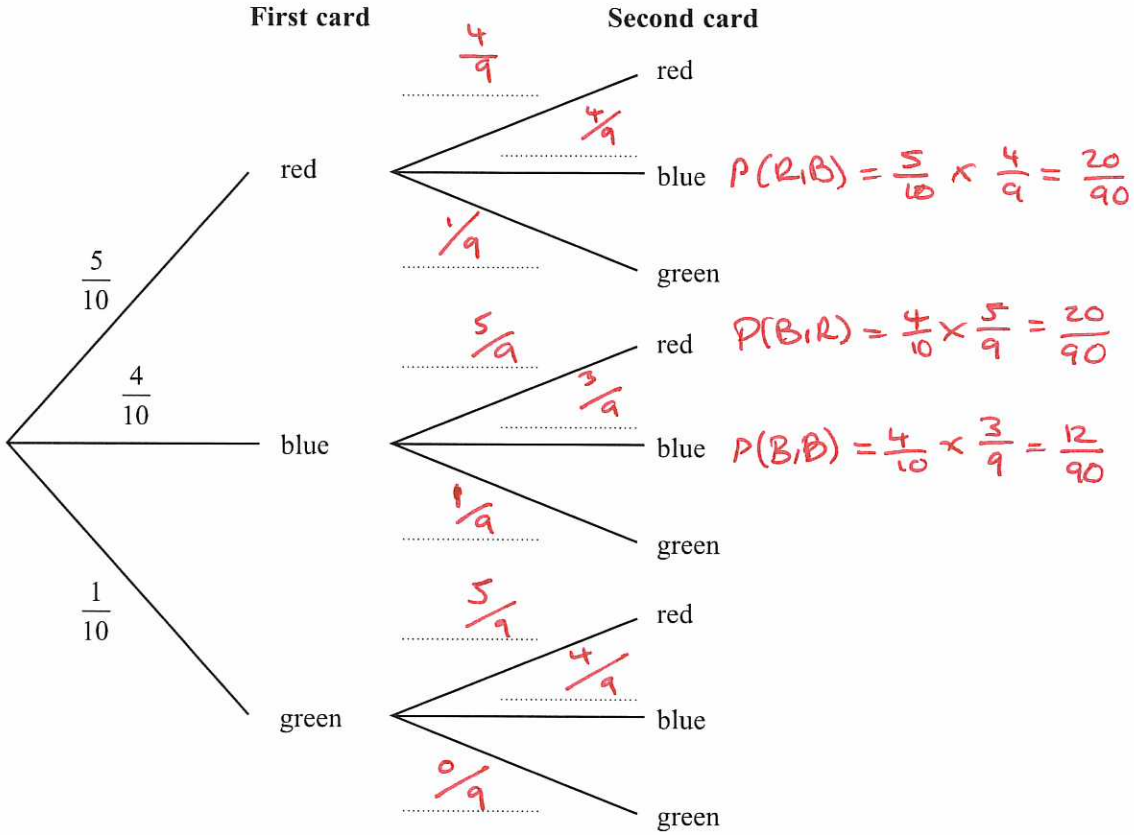
(Total for Question 7 is 5 marks)

Dependent Tree Diagrams

8 Felix has 10 cards.
There are 5 red cards, 4 blue cards and 1 green card.

Felix takes at random one of the cards.
He does not replace the card.
Felix then takes at random a second card.

(*a*) Complete the probability tree diagram.



(2)

(*b*) Work out the probability that Felix takes at least one blue card and no green card.

= P(B,B) + P(B,R) + P(R,B)

= 12/90 + 20/90 + 20/90

= 52/90

52/90

(3)

(Total for Question 8 is 5 marks)

Circle Theorems

9 In the diagram below, P and Q are points on a circle with centre O.

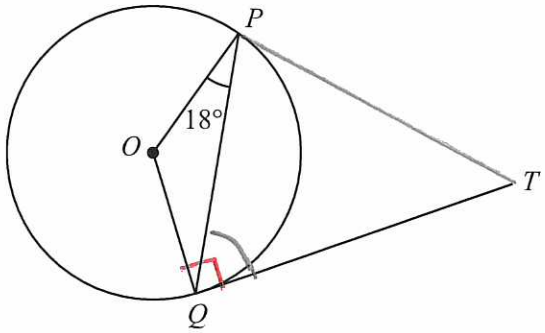


Diagram **NOT** accurately drawn

QT is a tangent to the circle.
Angle $OPQ = 18°$

Work out the size of angle PQT.
Give a reason for each stage of your working.

$O\hat{Q}T = 90°$

$O\hat{Q}P = 18°$

$\therefore P\hat{Q}T = 90° - 18°$

$= 72°$

Tangent meets radius at 90°.

Isosceles triangle (both OP and OQ are radii)

72 °

(Total for Question 9 is 3 marks)

10 The function f is such that

$$f(x) = \frac{3x-5}{4}$$

(*a*) Find f (–7)

$f(-7) = \frac{3(-7)-5}{4} = \frac{-21-5}{4} = \frac{-26}{4} = -6.5$

-6.5

.....

(1)

(*b*) Express the inverse function f^{-1} in the form $f^{-1}(x) = ...$

$f(x) = \frac{3x-5}{4}$

$y = \frac{3x-5}{4}$

(×4) $4y = 3x-5$

(+5) $4y+5 = 3x$

(÷3) $\frac{4y+5}{3} = x$

Notation $\frac{4x+5}{3} = f^{-1}(x)$

$f^{-1}(x) = \frac{4x+5}{3}$

.....

(2)

The function g is such that

$$g(x) = \sqrt{19-x}$$

(*c*) Find fg (3)

$g(3) = \sqrt{19-3} = \sqrt{16} = 4$

$\therefore fg(3) = f(4) = \frac{3(4)-5}{4} = \frac{12-5}{4} = \frac{7}{4}$

.....

(2)

(Total for Question 10 is 5 marks)

Index Laws

11 $\dfrac{8}{2^7} = 2^n$

(*a*) Find the value of *n*.

$8 = 2^3$ | $= \dfrac{2^3}{2^7}$

$a^m \div a^n = a^{m-n}$ | $= 2^{3-7} = 2^{-4}$

n = -4

(2)

$(13^{-6})^4 \times 13^5 = 13^k$

(*b*) Find the value of *k*.

$(a^m)^n = a^{m \times n}$ | $= 13^{-24} \times 13^5$

$a^m \times a^n = a^{m+n}$ | $= 13^{-24+5} = 13^{-19}$

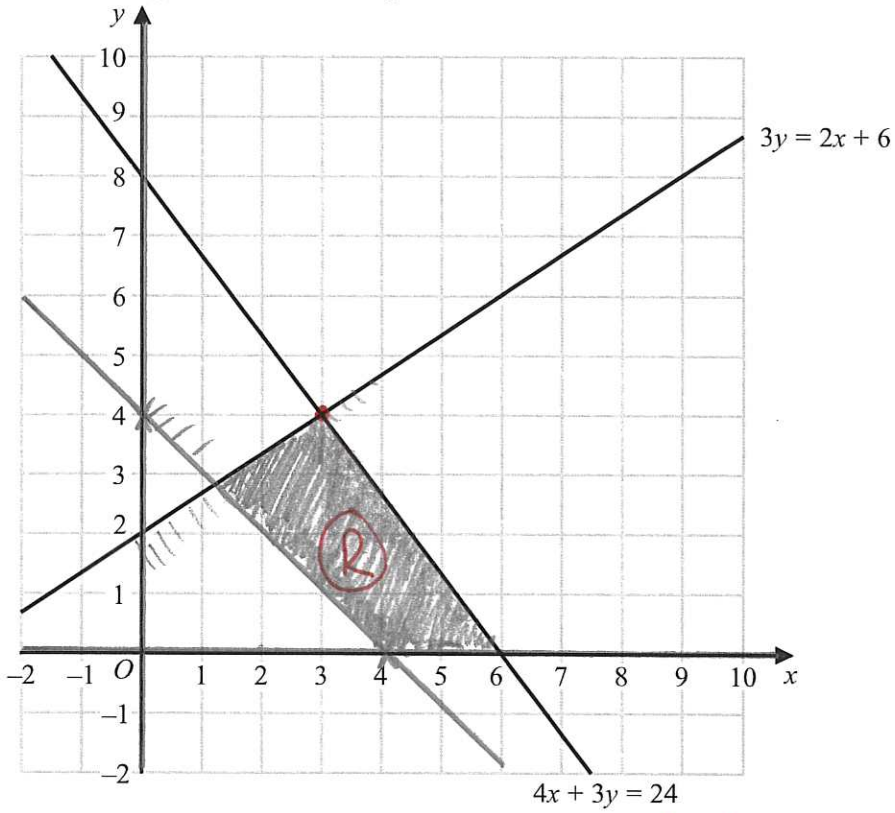
k = -19

(2)

(Total for Question 11 is 4 marks)

Graphical Inequalities

12 The diagram shows two straight lines drawn on a grid.



(*a*) Write down the solution of the simultaneous equations ← point of intersection

$$3y = 2x + 6$$
$$4x + 3y = 24$$

$x =$ 3

$y =$ 4

(1)

(*b*) Show, by shading on the grid, the region defined by all five of the inequalities

$x \geqslant 0$ $y \geqslant 0$ $x + y \geqslant 4$ $3y \leqslant 2x + 6$ $4x + 3y \leqslant 24$

Implicit function... find x and y intercepts

Label the region **R**.

(0,4) (4,0) (0,2) (-3,0) (6,0) (0,8)

DRAWN ALREADY.

(3)

(Total for Question 12 is 4 marks)

Vectors

13 The diagram shows parallelogram $ABCD$.

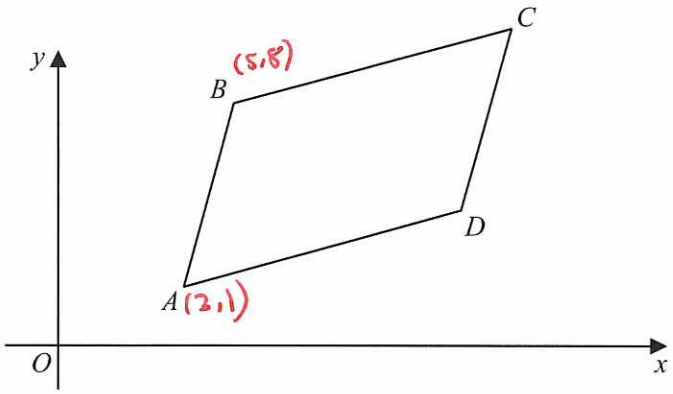


Diagram **NOT** accurately drawn

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$

The point B has coordinates (5, 8)

(*a*) Work out the coordinates of the point C.

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$ | $A = (x, y)$ $B = (5, 8)$

$\therefore$ | $A = (3, 1)$

$\overrightarrow{AC} = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$ | $A = (3, 1)$ $C = (p, q)$

$C = (13, 12)$

(13 , 12)

(3)

The point E has coordinates (63, 211)

(*b*) Use a vector method to prove that ABE is a straight line.

Ridiculous mark scheme

THIS IS STILL A VECTOR METHOD.

Gradient of AB | Two points: (3,1) and (5,8) (A, B)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ | $m = \frac{8-1}{5-3} = \frac{7}{2}$

Gradient of BE | Two points: (5,8) and (63,211)

$m = \frac{y_2 - y_1}{x_2 - x_1}$ | $m = \frac{211-8}{63-5} = \frac{203}{58} = \frac{7}{2}$

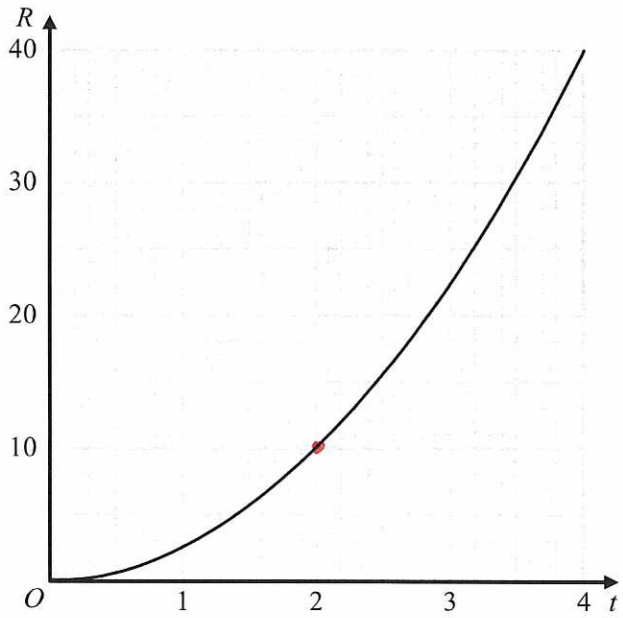
(2)

conclusion | Two vectors have same gradient and share a point B $\therefore$ Straight line.

(Total for Question 13 is 5 marks)

Direct Proportion

14 R is proportional to t^2
The graph shows the relationship between R and t for $0 \leqslant t \leqslant 4$



(*a*) Find a formula for R in terms of t.

when $t=2$, $R=10$

Direct proportion

$R \propto t^2$

$R = kt^2$

$t=2$, $R=10$

$10 = k(2)^2$

$10 = 4k$

$(\div 4)$

$\frac{5}{2} = k$

Formula for R

$R = \frac{5}{2}t^2$

$R = \frac{5}{2}t^2$

.....

(3)

Inverse Proportion

Given also that $R = \frac{8}{5x}$

(*b*) show that t is inversely proportional to $\sqrt{x}$ for $t > 0$

$R = \frac{8}{5x}$

$R = \frac{5}{2}t^2$ $\quad \frac{5}{2}t^2 = \frac{8}{5x}$

$(\times 2)$ $\quad 5t^2 = \frac{16}{5x}$

$(\div 5)$ $\quad t^2 = \frac{16}{25x}$

$\sqrt{\text{ANS}}$ $\quad t = \sqrt{\frac{16}{25x}}$

$\left(k = \sqrt{\frac{16}{25}}\right)$ $\quad t = \frac{k}{\sqrt{x}}$

Inverse Proportion $\quad t \propto \sqrt{x}$ □.

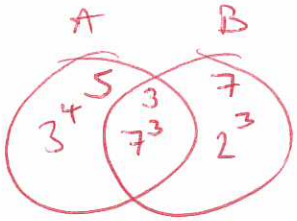
(2)

(Total for Question 14 is 5 marks)

HCF LCM Venn Diagrams

15 $A = 3^5 \times 5 \times 7^3$
$B = 2^3 \times 3 \times 7^4$

(*a*) (i) Find the Highest Common Factor (HCF) of A and B.



HCF = 3×7^3

(ii) Find the Lowest Common Multiple (LCM) of A and B.

LCM = $2^3 \times 3^5 \times 7^4 \times 5$

(2)

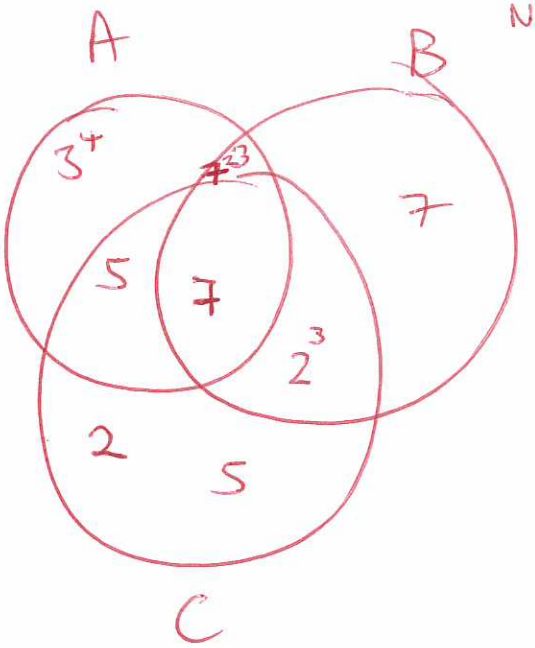
$A = 3^5 \times 5 \times 7^3$
$B = 2^3 \times 3 \times 7^4$
$C = 2^p \times 5^q \times 7^r$

HORRIBLE QUESTION

Given that
the HCF of B and C is $2^3 \times 7$ → 7 is a factor of all. 2^3 in B and C but not A
the LCM of A and C is $2^4 \times 3^5 \times 5^2 \times 7^3$ → 7^2 in A and B but 7' in A and B and C. → 5' in A

(*b*) find the value of p, the value of q and the value of r.

NOT in A → 3 in A and B



Now CHECK. $A = 3^5 \times 5 \times 7^3$ ✓
$B = 2^3 \times 3 \times 7^4$ → Needs a 7. ✓
$C = 2^p \times 5^q \times 7^r$
∴ $p = 4, q = 2, r = 1$

$p =$ 4
$q =$ 2
$r =$ 1

(2)

(Total for Question 15 is 4 marks)

Independent Events

16 Jack plays a game with two fair spinners, **A** and **B**.

Spinner **A** can land on the number 2 or 3 or 5 or 7.
Spinner **B** can land on the number 2 or 3 or 4 or 5 or 6.

Jack spins both spinners.
He wins the game if one spinner lands on an odd number **and** the other spinner lands on an even number.

Jack plays the game twice.
Work out the probability that Jack wins the game both times.

P(spinner A = odd) = $\frac{3}{4}$ P(spinner A = even) = $\frac{1}{4}$

P(spinner B = odd) = $\frac{2}{5}$ P(spinner B = even) = $\frac{3}{5}$

P(Jack wins) = $\left(\frac{3}{4} \times \frac{3}{5}\right) + \left(\frac{1}{4} \times \frac{2}{5}\right)$

= $\frac{9}{20} + \frac{2}{20} = \frac{11}{20}$

P(win, win) = $\frac{11}{20} \times \frac{11}{20}$

= $\frac{121}{400}$

$\frac{121}{400}$

.....

(Total for Question 16 is 4 marks)

Algebraic Fractions

17 Express $\frac{1}{9x^2-25} - \frac{1}{6x+10}$ as a single fraction in its simplest form.

cross multiply $\quad \frac{6x+10-(9x^2-25)}{(9x^2-25)(6x+10)}$

expand top $\quad = \frac{6x+10-9x^2+25}{(9x^2-25)(6x+10)}$

factorise bottom $\quad = \frac{6x-9x^2+35}{(3x+5)(3x-5)2(3x+5)}$

factorise top $\quad = \frac{(3x+5)(7-3x)}{(3x+5)(3x-5)2(3x+5)}$

simplify $\quad = \frac{7-3x}{2(3x-5)(3x+5)}$

$\frac{7-3x}{2(3x-5)(3x+5)}$

(3)

(Total for Question 17 is 3 marks)

Surds

18 (*a*) Show that $\sqrt{45}+\sqrt{20}=5\sqrt{5}$

Show your working clearly.

$\sqrt{45} = \sqrt{9}\sqrt{5}$

$= 3\sqrt{5}$

$\sqrt{20} = \sqrt{4}\sqrt{5}$

$= 2\sqrt{5}$

$3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$ □.

(2)

(*b*) Express $\frac{2}{\sqrt{3}-1}$ in the form $p + \sqrt{q}$

Rationalising

where p and q are integers.

Show your working clearly.

Rationalise $\times \left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$\frac{2}{\sqrt{3}-1}$

$= \frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{2(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{2\sqrt{3}+2}{3+\sqrt{3}-\sqrt{3}-1}$

$= \frac{2\sqrt{3}+2}{2}$

$= \sqrt{3}+1 \quad = 1+\sqrt{3}$

$1+\sqrt{3}$

.....

(2)

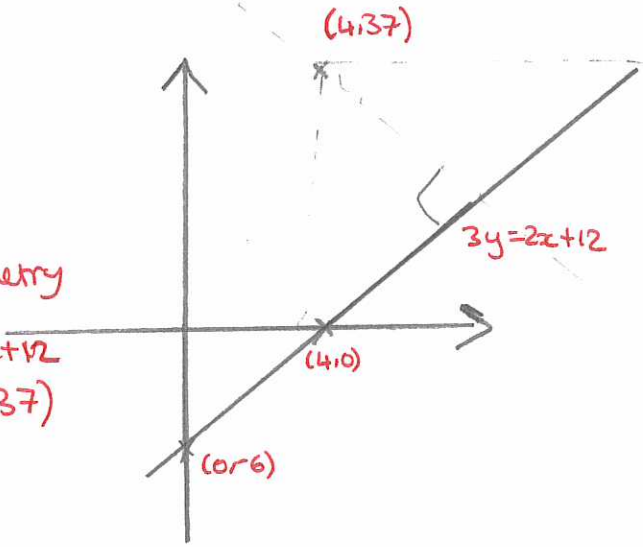
(Total for Question 18 is 4 marks)

Perpendicular Lines

19 ABC is an isosceles triangle such that

$AB = AC$
A has coordinates (4, 37)
B and C lie on the line with equation $3y = 2x + 12$ (0,4) and (-6,0)

Find an equation of the line of symmetry of triangle ABC.
Give your answer in the form $px + qy = r$ where p, q and r are integers.
Show clear algebraic working.



Sketch

$3y = 2x + 12$

$(\div 3)$ $y = \frac{2}{3}x + 4$

$\therefore m = \frac{2}{3}$

Sketch

Sketch shows a line of symmetry will be perpendicular to $3y=2x+12$ through (4,37)

$m_1 \times m_2 = -1$

$\therefore m_2 = -\frac{3}{2}$

$y = mx + c$
(4,37)

$y = -\frac{3}{2}x + c$

$37 = -\frac{3}{2}(4) + c$

$37 = -\frac{12}{2} + c$

$37 = -6 + c$

$(+6)$ $43 = c$

$y = mx + c$ $y = -\frac{3}{2}x + 43$

$(\times 2)$ $2y = -3x + 86$

$(+3x)$ $3x + 2y = 86$

$Px + Qy = R$

$3x + 2y = 86$

(Total for Question 19 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS