MS REY

Q13

GCSE Mathematics

Practice Tests: Set 7

Paper 3H (Calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

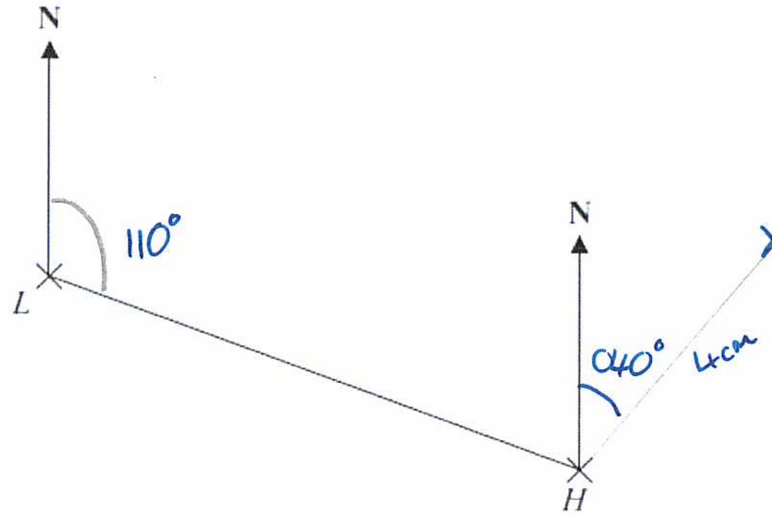
Bearings and Scales

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. The diagram shows the position of a lighthouse *L* and a harbour *H*.



The scale of the diagram is 1 cm represents 5 km.

(a) Work out the real distance between *L* and *H*.

×7.5 ↓ 1cm : 5km ↓ ×7.5

7.5cm : 37.5km

MARK SCHEME IS WRONG

37.5 km

(1)

(b) Measure the bearing of *H* from *L*.

START AT L.

110 °

(1)

A boat *B* is 20 km from *H* on a bearing of 040°.

(c) On the diagram, mark the position of boat *B* with a cross (×).
Label it *B*.

Start at H.

×4 ↓ 1cm : 5km ↑ ÷4

4cm : 20km

(2)

(Total for Question 1 is 4 marks)

2. A mixture of sugar and salt is in the ratio 3 : 2
The weight of the mixture is 150 grams.

(a) Calculate the weight of sugar and the weight of salt in the mixture.

Sugar : Salt | 3+2 = 5 parts
5parts = 150g
(÷~~35~~) | 1 part = 30g
Sugar : Salt | 3 : 2
(×20) | 90 : 60

Sugar ....90....g

Salt ....60....g

(3)

30 grams of sugar and 10 grams of salt are added to the mixture.

(b) Calculate the ratio of sugar to salt in the new mixture.

Sugar : Salt | 90 : 60
(+30) (+10) | 120 : 70
(÷10) | 12 : 7

....12 : 7....

(2)

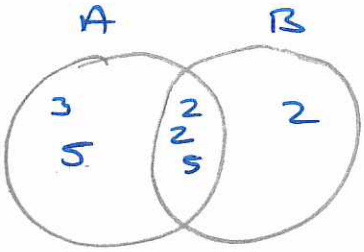
(Total for Question 2 is 5 marks)

LCM HCF Venn Diagrams

3. $A = 2^2 \times 3 \times 5^2$

$B = 2^3 \times 5$

(a) Find the Highest Common Factor (HCF) of A and B.



$A = \cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{5} \times \cancel{5}$

$B = \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{5}$

$HCF = 2 \times 2 \times 5$

$= \underline{20}$

....

(1)

(b) Find the Lowest Common Multiple (LCM) of A and B.

$LCM = 2 \times 2 \times 5 \times 3 \times 5 \times 2$

$= \underline{\underline{600}}$

600
....

(2)

(Total for Question 3 is 3 marks)

Box Plots

4. Here are the ages, in years, of 15 women at West Ribble Tennis Club.

16, 18, 18, 20, 25, 25, 27, 28, 30, 35, 38, 42, 45, 46, 50

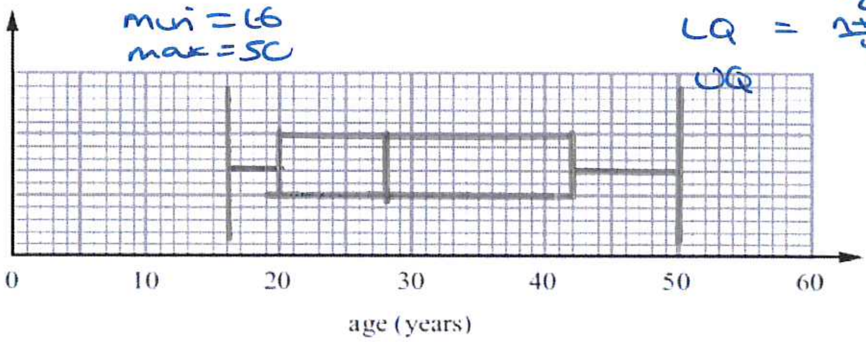
(a) On the grid, draw a box plot for this information.

Median $= \frac{n+1}{2} = \frac{16}{2} = 8^{th}$ term $= 28$

LQ $= \frac{n+1}{4} = \frac{16}{4} = 4^{th}$ term $= 20$

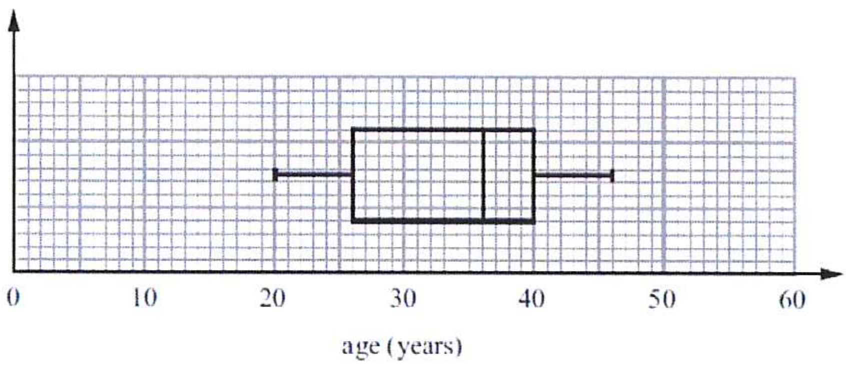
UQ $= 12^{th}$ term $= 42$

min = 16
max = 50



(3)

The box plot below shows the distribution of the ages of the men at West Ribble Tennis Club.



(b) Use the box plots to compare the distributions of the ages of these women and the distributions of the ages of these men.

On average women are younger than ~~[illegible]~~ at the club (smaller median)

(2)

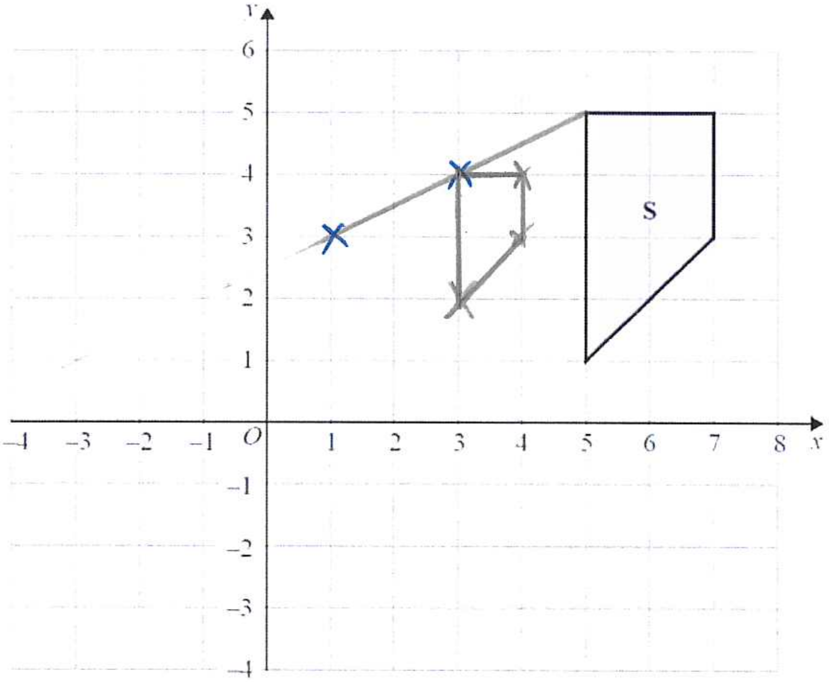
The spread of men's ages is less than women's (smaller IQR).

(Total for Question 4 is 5 marks)

Transformations

vectors: $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $SF = \frac{1}{2}$ $\frac{1}{2}\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

5.



Enlarge shape **S** with scale factor $\frac{1}{2}$ and centre (1, 3).

(Total for Question 5 is 2 marks)

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6. Given that, for all values of x,

Expanding Brackets

$$6x^3 + 7x^2 - 56x + 48 = (2x^2 + kx - 12)(3x - 4), \text{ where } k \text{ is a constant,}$$

find the value of k.

$(2x^2+kx-12)(3x-4)$

$= 6x^3 - 8x^2 + 3kx^2 - 4kx - 36x + 48$

collect $= 6x^3 + (3k-8)x^2 + (-36-4k)x + 48$

compare $6x^3 + 7x^2 - 56x + 48$

x^2 and x co-efficients: $3k - 8 = 7$

(+8) $3k = 15$

(÷3) $k = 5$

$k =$ 5

(Total for Question 6 is 2 marks)

Area of 2D Shapes
Pythagoras

7.

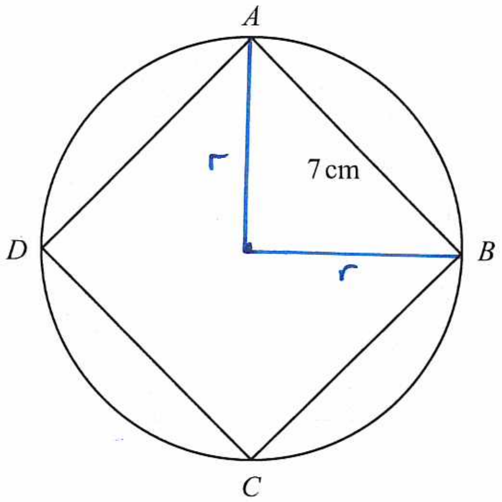
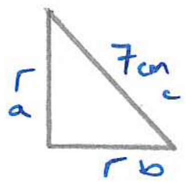


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle.
ABCD is a square of side 7 cm.

Work out the total area of the shaded regions.
Give your answer correct to the nearest whole number.

Area of circle $= \pi r^2$



But we don't have r...

Pythagoras: $r^2 + r^2 = 7^2$

$2r^2 = 49$

$r^2 = 49/2$

$r = \sqrt{\frac{49}{2}}$

Area of circle $= \pi r^2$

Area $= \pi \left(\sqrt{\frac{49}{2}}\right)^2 = \frac{49}{2}\pi \text{ cm}^2$

Area of square $= L \times L$

Area $= 7\text{cm} \times 7\text{cm} = 49\text{cm}^2$

Shaded = Total − Non shaded

Shaded = Area of circle − Area of square

Shaded $= \frac{49}{2}\pi \text{ cm}^2 - 49\text{cm}^2$

$= 27.96902001$

$= 28$ (nearest whole)

28 .... cm^2

(Total for Question 7 is 5 marks)

Repeated Percentage Change

8. Danielle invested £2800 for n years in a savings account.
She was paid 2.5% per annum compound interest.
The interest is paid into the account at the end of each year.
At the end of n years, the amount of money in the savings account is greater than £3000 for the first time.

Work out the value of n.

Start × multiplier^t = End | $2800 \times 1.025^n = 3000$
multiplier = 1 + x% = 1.025

Try $n=1$: $2800 \times 1.025^1 = 2870$
$n=2$: ... $= 2941.75$
$n=3$: $= 3015.29$

Conclusion: $n=3$

$n=3$

(Total for Question 8 is 3 marks)

Algebraic Proof

9. n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n - 2)^2$ is always an even number.

Expand	$n^2-2-(n-2)(n-2)$
	$=n^2-2-[n^2-2n-2n+4]$
collect	$=n^2-2-[n^2-4n+4]$
expand	$=n^2-2-n^2+4n-4$
collect	$=4n-6$
factorise	$=2(2n-3)$
Conclusion	$2m$ is even $\therefore 2(2n-3)$ is even $\square$.

(Total for Question 9 is 2 marks)

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10. Make e the subject of $k = \sqrt{\dfrac{5m+2e}{3e}}$

Rearranging Formula (complex)

$(\text{ANS})^2$	$k^2 = \dfrac{5m+2e}{3e}$
$(\times 3e)$	$3k^2e = 5m+2e$
$(-2e)$	$3k^2e-2e = 5m$
factorise	$e(3k^2-2) = 5m$
$(\div(3k^2-2))$	$e = \dfrac{5m}{3k^2-2}$

$e = \dfrac{5m}{3k^2-2}$

(Total for Question 10 is 4 marks)

Iteration

11. (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

$x^3+4x=1$

(-1) $x^3+4x-1=0$

let $f(x) = x^3+4x-1$

$f(0) = (0)^3+4(0)-1 = -1$

$f(1) = (1)^3+4(1)-1 = 4$

conclusion: change of sign rule indicates solution lies between inputs 0 and 1 $\square$.

(2)

(b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

$x^3+4x=1$

$(-x^3)$ $4x = 1-x^3$

$(\div 4)$ $x = \frac{1}{4} - \frac{x^3}{4}$ $\square$.

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^{\,3}}{4}$ twice,

to find an estimate for the solution of $x^3 + 4x = 1$

$x_0 = 0 = \text{ANS}$

$x_1 = \frac{1}{4} - \frac{(\text{ANS})^3}{4}$

$= 0.25$

$x_2 = 0.24609375$

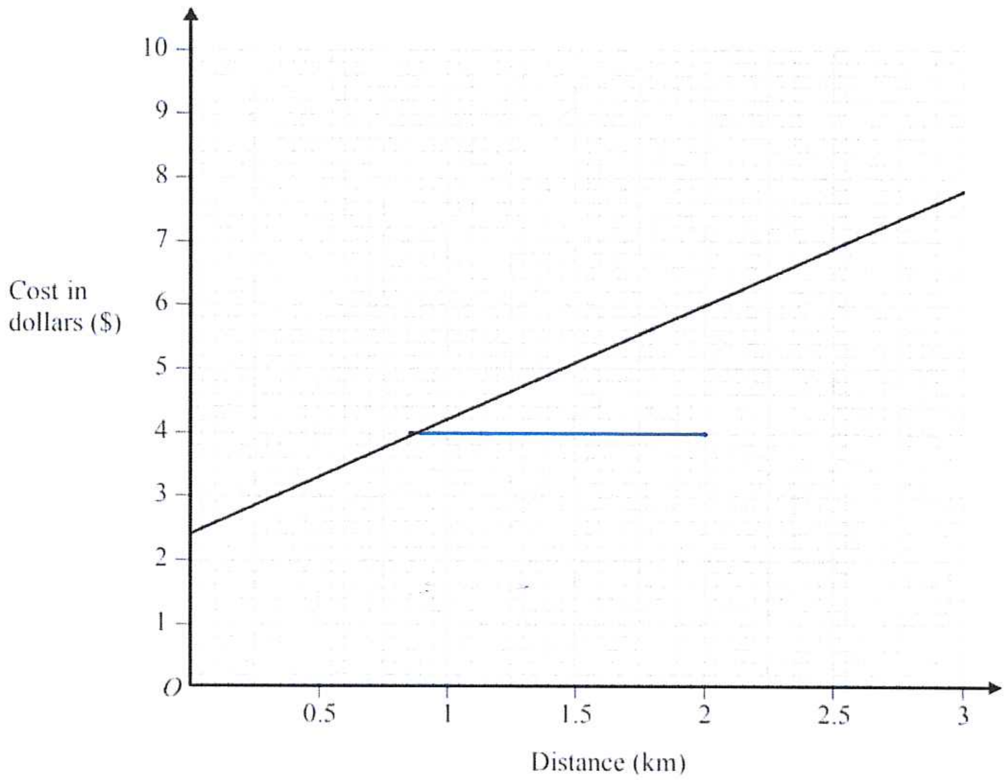
0.24609375 ....

(3)

(Total for Question 11 is 6 marks)

Interpreting Real Life Graphs

12.



The graph gives information about the costs of taxi journeys of different distances.
The cost of a taxi journey consists of a fixed initial charge and a charge per km.

(a) Give an interpretation of the intercept of the graph on the y-axis.

The initial charge of a taxi journey is $2.50.

(1)

(b) Give an interpretation of the gradient of the graph.

The charge per km after entering the taxi

(1)

(Total for Question 12 is 2 marks)

Inverse Functions

NOTE: Mark scheme is WRONG

13. $f(x) = \dfrac{4}{x-3}$ $\qquad$ $g(x) = \dfrac{x-2}{x}$

(*a*) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

	$y = \dfrac{4}{x-3}$
$(\times(x-3))$	$y(x-3) = 4$
Expand	$xy - 3y = 4$
$(+3y)$	$xy = 4 + 3y$
$(\div y)$	$x = \dfrac{4+3y}{y}$
Notation	$f^{-1}(x) = \dfrac{4+3x}{x}$

$f^{-1}(x) = \dfrac{4+3x}{x}$

(3)

(*b*) Solve $\quad fg(a) = 1$

Composite Functions

Just use x instead of a...

You must show your working.

$f(g(a))$	$= \dfrac{4}{\frac{x-2}{x} - 3}$
combine denominator	$= \dfrac{4}{\frac{x-2-3x}{x}}$
collect	$= \dfrac{4}{\frac{-2x-2}{x}} = 4 \div \dfrac{-2x-2}{x}$
KCF	$= 4 \times \dfrac{x}{-2x-2}$
$f(g(a))$	$= \dfrac{4x}{-2x-2}$

$a = \ldots\ldots\ldots\ldots$

(3)

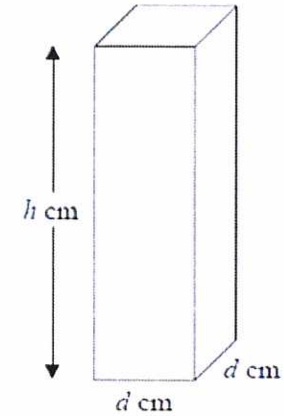
(Total for Question 13 is 6 marks)

$f(g(a)) = 1$	$\dfrac{4x}{-2x-2} = 1$
$\times(-2x-2)$	$4x = -2x - 2$
$(+2x)$	$6x = -2$
$(\div 6)$	$x = -\dfrac{2}{6} = -\dfrac{1}{3}$

Density Mass Volume
Bounds

$\dfrac{M}{D \mid V}$

14. Here is a solid bar made of metal.



The bar is in the shape of a cuboid.
The height of the bar is h cm.
The base of the bar is a square of side d cm.

The mass of the bar is M kg.

$d = 8.3$ correct to 1 decimal place.
$M = 13.91$ correct to 2 decimal places.
$h = 84$ correct to the nearest whole number.

Find the value of the density of the metal to an appropriate degree of accuracy.
Give your answer in g/cm^3.

You must explain why your answer is to an appropriate degree of accuracy.

d error: $8.3 \xrightarrow{+0.05} 8.35$ d_{max}, $8.3 \xrightarrow{-0.05} 8.25$ d_{min} $\quad 0.1 \div 2 = 0.05$

M error: $13.91 \xrightarrow{+0.005} 13.915$ M_{max}, $13.91 \xrightarrow{-0.005} 13.905$ M_{min} $\quad 0.01 \div 2 = 0.005$

h error: $84 \xrightarrow{+0.5} 84.5$ h_{max}, $84 \xrightarrow{-0.5} 83.5$ h_{min} $\quad 1 \div 2 = 0.5$

$D_{max} = \dfrac{M_{max}}{V_{min}}$
$(V_{min} = h_{min} \times d_{min} \times d_{min})$
convert to g/cm^3

$$D_{max} = \frac{13.915\text{kg}}{83.5\text{cm} \times 8.25\text{cm} \times 8.25\text{cm}} = 0.0024484\ldots\text{kg/cm}^3$$
$$= 2.4484\ldots \text{g/cm}^3$$

$D_{min} = \dfrac{M_{min}}{V_{max}}$
$(V_{max} = h_{max} \times d_{max} \times d_{max})$
convert to g/cm^3

$$D_{min} = \frac{13.905\text{kg}}{84.5\text{cm} \times 8.35\text{cm} \times 8.35\text{cm}} = 0.0023601\ldots\text{kg/cm}^3$$
$$= 2.3601\ldots \text{g/cm}^3$$

(Total for Question 14 is 5 marks)

Round to "appropriate accuracy"

$D_{max} = D_{min}$ when rounded to 2s.f
$\therefore D = 2.4$ (2s.f) g/cm^3

Ratio and Proportion

15. 60 apples are shared between Abbie, Betty and Carol in the ratios 1 : 3 : x, where $x > 3$.

The number of apples in Carol's share is 18 more than the number of apples in Betty's share.

Find the value of x.

Attempt 1

A : B : C	1 : 3 : x	Total
let $x = 4$	1 : 3 : 4	8
(×2)	2 : 6 : 8	16

X

Won't work since sum of parts is a multiple of 8 and 60 is NOT a multiple of 8.

x = ...

(Total for Question 15 is 4 marks)

Attempt 2

A : B : C	1 : 3 : x	Total
let $x = 5$	1 : 3 : 5	9
(×2)	2 : 6 : 10	18

X

WRONG: Same as before

Attempt 3

A : B : C	1 : 3 : x		Total
let $x = 6$	1 : 3 : 6	d = 3	10
(×2)	2 : 6 : 12	d = 6	
...	3 : 9 : 18	d = 9	
...	4 : 12 : 24	d = 12	
...	5 : 15 : 30	d = 15	
	6 : 18 : 36	d = 18	Total = 60

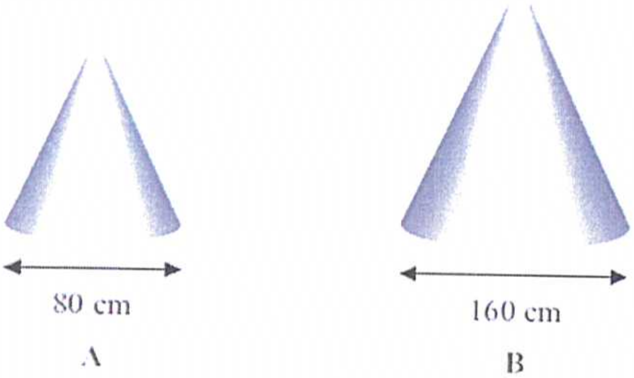
WANT: Difference of B and C = 18

$\therefore x = 6$

LSF ASF VSF

16. Ali has two solid cones made from the same type of metal.

Diagram **NOT** accurately drawn



The two solid cones are mathematically similar.
The base of cone **A** is a circle with diameter 80 cm.
The base of cone **B** is a circle with diameter 160 cm.
Ali uses 80 m*l* of paint to paint cone **A**.
Ali is going to paint cone **B**.

Painting = Coating an Area

(a) Work out how much paint, in m*l*, he will need.

LSF A : B | 80 : 160
(÷80) | 1 : 2
ASF = (LSF)^2 | 1 : 4
(×80) | 80 : 320

......320...... m*l*
(2)

The volume of cone **A** is 171 700 cm^3.

(b) Work out the volume of cone **B**.

LSF A : B | 1 : 2
VSF = (LSF)^3 | 1 : 8
↓ ×171,700
(×171,700) | 171,700 : 1,373,600

......1373,000...... cm^3
(3)

(Total for Question 16 is 5 marks)

Vectors

17.

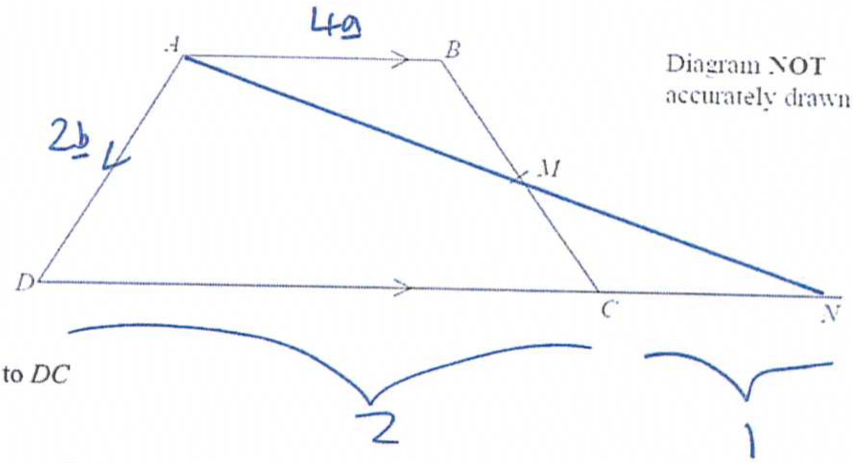


Diagram NOT accurately drawn

AB is parallel to DC

$DC = 2AB$

M is the midpoint of BC

$\overrightarrow{AD} = 2\mathbf{b}$

$\overrightarrow{AB} = 4\mathbf{a}$

(*a*) Find $\overrightarrow{BM}$ in terms of **a** and **b**.

Give your answer in its simplest form.

$\overrightarrow{DC} = 2\overrightarrow{AB}$

$$\overrightarrow{BM} = \tfrac{1}{2}\overrightarrow{BC}$$
$$= \tfrac{1}{2}(\overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC})$$
$$= \tfrac{1}{2}(-4\underline{a} + 2\underline{b} + 2(4\underline{a}))$$
$$= \tfrac{1}{2}(-4\underline{a} + 2\underline{b} + 8\underline{a})$$
$$= \tfrac{1}{2}(+4\underline{a} + 2\underline{b})$$

....

$$= +2\underline{a} + \underline{b}$$

(2)

N is the point such that DCN is a straight line and $DC : CN = 2 : 1$

(*b*) Show that AMN is a straight line.

$\overrightarrow{AM}$

$\overrightarrow{BM}$ using (a)

$$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM}$$
$$= 4a + +2\underline{a} + b$$
$$= 6\underline{a} + \underline{b}$$

$\overrightarrow{MN}$

$\overrightarrow{MC} = \overrightarrow{BM}$, $\overrightarrow{CN} = \tfrac{1}{2}\overrightarrow{DC}$

$$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CN}$$
$$= \overrightarrow{BM} + \tfrac{1}{2}\overrightarrow{DC}$$
$$= +2\underline{a} + \underline{b} + \tfrac{1}{2}(8a)$$
$$= 6\underline{a} + \underline{b}$$

(2)

(Total for Question 17 is 4 marks)

$\overrightarrow{AM} = \lambda\overrightarrow{MN}$ $(\lambda = 1)$

$\therefore$ $\overrightarrow{AM}$ and $\overrightarrow{MN}$ parallel

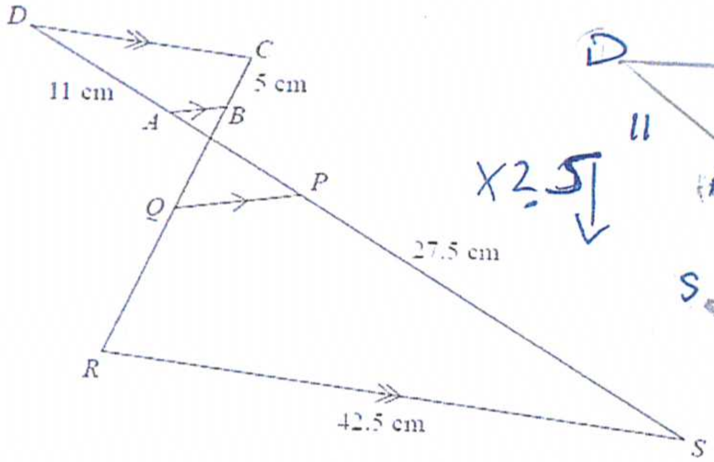
Also $\overrightarrow{AM}$ and $\overrightarrow{MN}$ share a point (M)

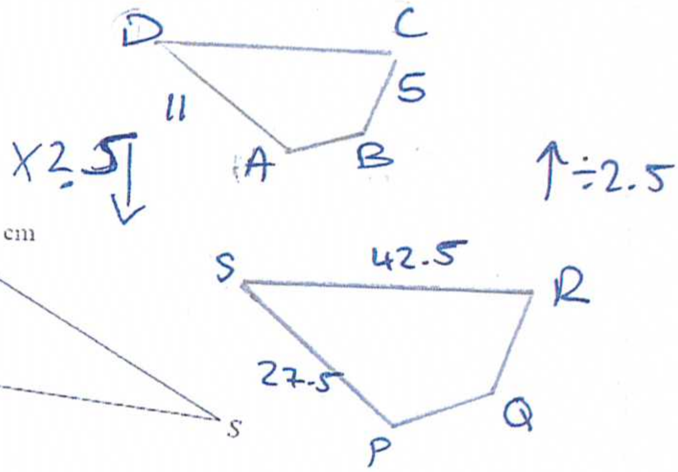
$\therefore$ AMN is a straight line. $\square$

Similar Shapes

18. In the diagram, *DAPS* and *CBQR* are straight lines.
AB is parallel to *QP* and *DC* is parallel to *RS*.
$AD = 11$ cm, $BC = 5$ cm, $PS = 27.5$ cm and $RS = 42.5$ cm.

Draw similar shapes...





Quadrilateral *ABCD* is similar to quadrilateral *PQRS*.

(a) Work out the length of *RQ*.

Scale factor: $\frac{27.5}{11} = 2.5$

$\therefore RQ = 2.5 \times 5\text{cm}$

$= 12.5\text{cm}$

12.5cm .... cm

(2)

(b) Work out the length of *CD*.

Scale factor $= 2.5$

$\therefore CD = 42.5 \div 2.5$

$= 17$

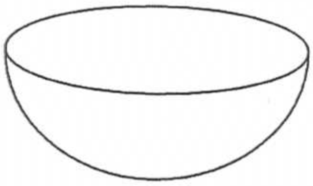
17cm .... cm

(2)

(Total for Question 18 is 4 marks)

Volume and Surface Area of spheres

19. The diagram shows a solid hemisphere.



Surface area of a sphere $= 4\pi r^2$
Volume of a sphere $= \dfrac{4}{3}\pi r^3$

The hemisphere has a **total** surface area of $\dfrac{16}{3}\pi$ cm^2

The hemisphere has a volume of $k\pi$ cm^3
Find the value of k.

Surface area (total) $=$ Curved surface area + flat circle surface

$= \dfrac{4\pi r^2}{2} + \pi r^2$

$= \dfrac{2\pi r^2}{\cancel{2}} + \pi r^2$

$= 3\pi r^2$

Given total Surface area $= \frac{16}{3}\pi$ $\quad \therefore 3\pi r^2 = \frac{16}{3}\pi$

$(\div \pi)$ $\quad \therefore 3r^2 = \frac{16}{3}$

$(\div 3)$ $\quad \therefore r^2 = \frac{16}{9}$

$\sqrt{\text{ANS}}$ $\quad \therefore r = \frac{4}{3}$

Volume $= \frac{4}{3}\pi r^3$ $\quad$ volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi\left(\frac{4}{3}\right)^3 = \frac{128}{81}\pi$

$V = K\pi$

....

(Total for Question 19 is 4 marks) $\therefore K = \dfrac{128}{81}$

Advanced Trig

20. *ABC* is a triangle.

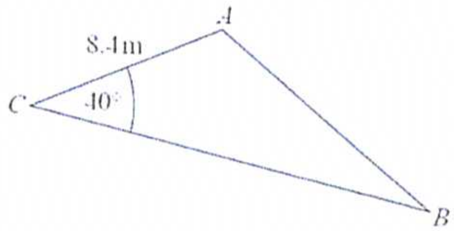
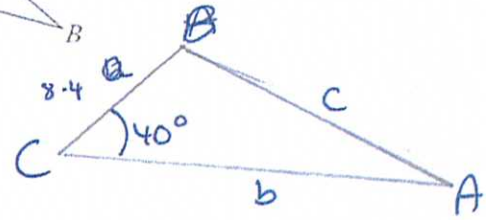


Diagram **NOT** accurately drawn

$AC = 8.4$ m
Angle $ACB = 40°$



The area of the triangle = 100 m^2.

Work out the length of *AB*.
Give your answer correct to 3 significant figures.
You must show all your working.

Area $= \frac{1}{2}ab\sin C$ | $100 = \frac{1}{2}(8.4)b\sin(40)$

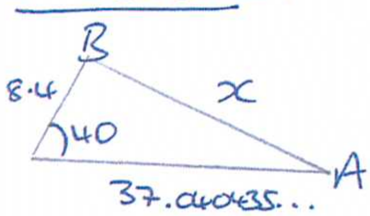
$(\times 2)$ | $200 = 8.4b\sin 40$

$(\div 8.4)$ | $\frac{500}{21} = b\sin 40$

$(\div \sin(40))$ | $37.0410435 = b$

$37.040435 = ANS.$

Cosine Rule: | $x^2 = 8.4^2 + ANS^2 - 2(8.4)(ANS)\cos(40)$



$x^2 = 965.897...$

$a^2 = b^2 + c^2 - 2bc\cos A$

$\sqrt{ANS}$ | $x = 31.1$ m $(3s.f)$

31.1 m

(Total for Question 20 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS