

GCSE Mathematics Practice Tests: Set 6

Paper 2H (Calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**



Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

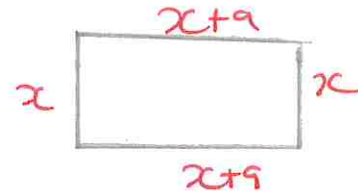
Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. The width of a rectangle is a whole number of centimetres.

The length of the rectangle is 9 cm longer than its width.
The perimeter of the rectangle is less than 200 cm.



Find the greatest possible width of the rectangle.

Perimeter < 200	$x + x + 9 + x + x + 9 < 200$
collect	$4x + 18 < 200$
(-18)	$4x < 182$
$(\div 4)$	$x < 45.5$
x is a whole number	$x = 45$ (Greatest value)

..... cm
(Total 4 marks)

2. A rugby team played six games.
The mean score for the six games is 14.5

missing mean

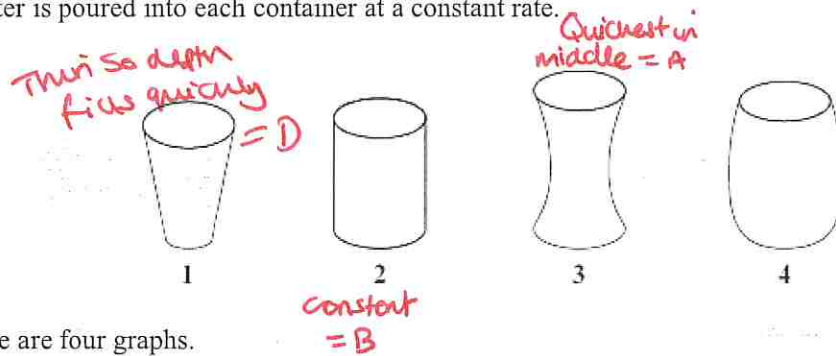
The rugby team played one more game.
The mean score for all seven games is 16

Work out the number of points the team scored in the seventh game.

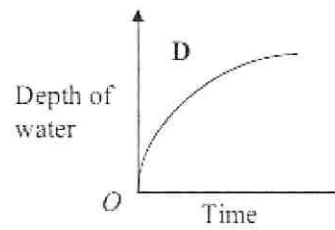
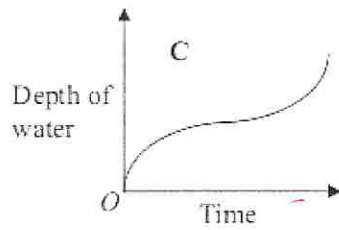
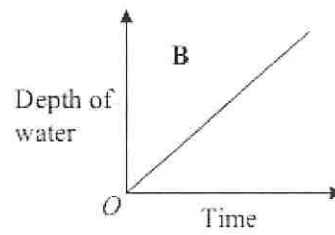
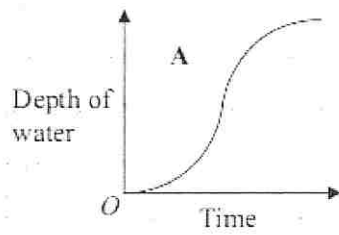
first 6 games total	$6 \times 14.5 = 87$
mean of 7 games formula (=16)	$= \frac{87 + x}{7} = 16$
$(\times 7)$	$87 + x = 112$
(-87)	$x = 25$

..... 25 points
(Total 2 marks)

3. Here are four containers.
Water is poured into each container at a constant rate.



Here are four graphs.
The graphs show how the depth of the water in each container changes with time.



Match each graph with the correct container.

- A and **3**
- B and **2**
- C and **4**
- D and **1**

(Total 2 marks)

Pythagoras / SƠ CẤP TỌA
with bearings

4. The diagram shows the positions of three turbines A, B and C.

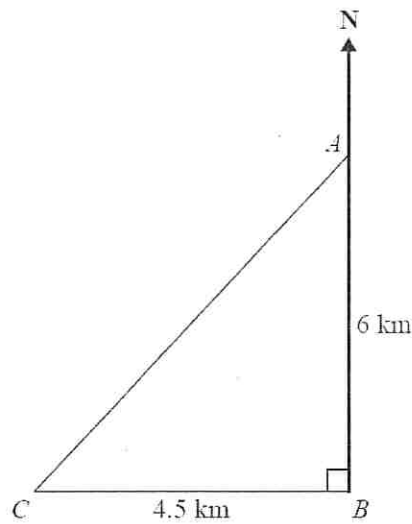


Diagram NOT accurately drawn

A is 6 km due north of turbine B.
C is 4.5 km due west of turbine B.

- (a) Calculate the distance AC.

Pythagoras

$$a^2 + b^2 = c^2$$

$$6^2 + 4.5^2 = 56.25 = c^2$$

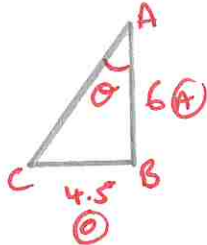
ANS

$$7.5 = c$$

..... 7.5 km
(3)

- (b) Calculate the bearing of C from A.
Give your answer correct to the nearest degree.

$$\text{Bearing} = 180^\circ + \theta^\circ \dots$$



shift tan

SƠ CẤP TỌA

$$\tan \theta = \frac{O}{A}$$

$$\tan \theta = \frac{4.5}{6}$$

$$\theta = 36.869 \dots = \text{ANS}$$

..... 217.

$$\text{Bearing} = 180^\circ + \theta^\circ$$

$$\text{Bearing} = 180 + \text{ANS}$$

$$= 217 \text{ (nearest degree)}$$

(Total 7 marks)

Volume of prisms

5. The diagram shows a prism.

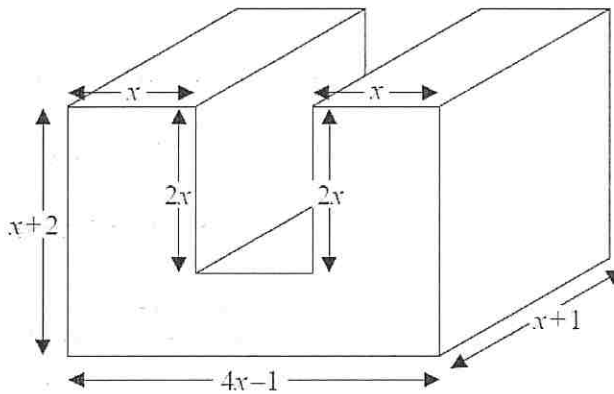


Diagram NOT accurately drawn

All measurements are in centimetres.
All corners are right angles.

Find an expression, in terms of x , for the volume, in cm^3 , of the prism.
You must show your working.
Give your answer in its simplest form.

Cross-sectional area of front face

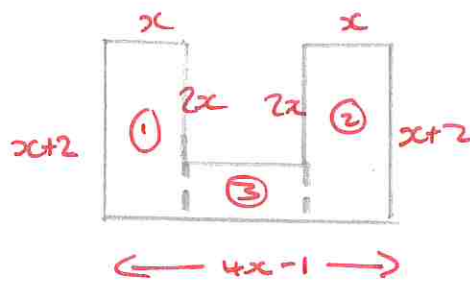
Volume = CSA \times length (depth)

CSA is a compound shape

③ length = $4x-1-x-x$
= $2x-1$

width = $x+2-2x$
= $2-x$

Volume = $(9x-2) \times$ length (depth)



① = $x(x+2) = x^2+2x$

② = $x(x+2) = x^2+2x$

③ = $lw = (2x-1)(2-x)$
= $4x-2x^2-2+x$

Total = $x^2+2x+x^2+2x+4x-2x^2+x-2$

CSA = $9x-2$

Volume = $(9x-2)(x+1)$
= $9x^2+9x-2x-2$
= $9x^2+7x-2$

(Total 4 marks)

Pythagoras

6. The diagram shows a triangle DEF inside a rectangle $ABCD$.

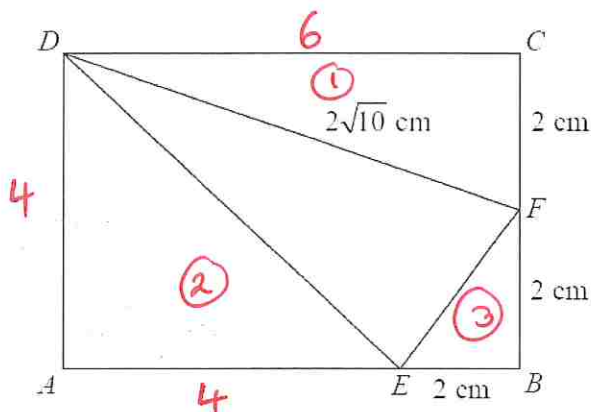
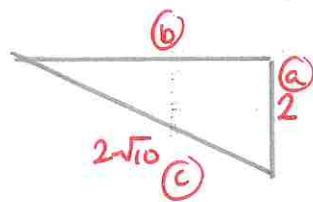


Diagram NOT accurately drawn

We are going to take the three triangle areas away from the total rectangle
 \therefore we need l and w of rectangle.

Show that the area of triangle DEF is 8 cm^2 .
 You must show all your working.



$$\sqrt{10} \times \sqrt{10} = 10$$

(-4)

$\sqrt{\text{ANS}}$

Pythagoras: $a^2 + b^2 = c^2$

$$2^2 + b^2 = (2\sqrt{10})^2$$

$$4 + b^2 = 2 \times \sqrt{10} \times 2 \times \sqrt{10}$$

$$4 + b^2 = 4 \times 10$$

$$4 + b^2 = 40$$

$$b^2 = 36$$

$$b = 6$$

Just use your calculator!

$$\text{Area } \textcircled{1} = \frac{b \times h}{2}$$

$$\textcircled{1} A = \frac{6 \times 2}{2} = 6 \text{ cm}^2$$

$$\text{Area } \textcircled{2} = \frac{b \times h}{2}$$

$$\textcircled{2} A = \frac{4 \times 4}{2} = 8 \text{ cm}^2$$

$$\text{Area } \textcircled{3} = \frac{b \times h}{2}$$

$$\textcircled{3} A = \frac{2 \times 2}{2} = 2 \text{ cm}^2$$

(Total 4 marks)

Area of Rectangle

$$A = 4 \times 6 = 24 \text{ cm}^2$$

Area of DEF

$$\begin{aligned} A &= 24 \text{ cm}^2 - 6 \text{ cm}^2 - 8 \text{ cm}^2 - 2 \text{ cm}^2 \\ &= \underline{\underline{8 \text{ cm}^2}} \quad \square \end{aligned}$$

Bounds

7. Jarek uses the formula

$$\text{Area} = \frac{1}{2} ab \sin C$$

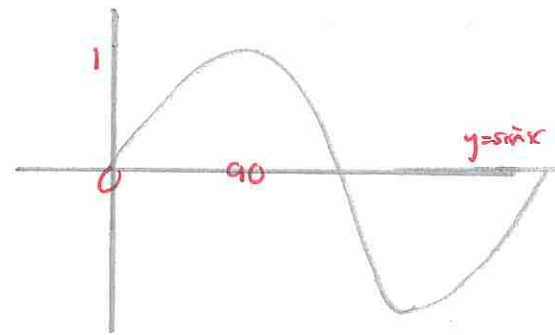
to work out the area of a triangle.

For this triangle,

$a = 7.8$ cm correct to the nearest mm.

$b = 5.2$ cm correct to the nearest mm.

$C = 63^\circ$ correct to the nearest degree.



Calculate the lower bound for the area of the triangle.

a error

$$7.8 \begin{array}{l} +0.05 \\ \hline 7.85 \text{ } a_{\max} \\ -0.05 \\ \hline 7.75 \text{ cm } a_{\min} \end{array} \quad 0.1 \div 2 = 0.05$$

b error

$$5.2 \begin{array}{l} +0.05 \\ \hline 5.25 \text{ } b_{\max} \\ -0.05 \\ \hline 5.15 \text{ } b_{\min} \end{array} \quad 0.1 \div 2 = 0.05$$

c error

$$63 \begin{array}{l} +0.5 \\ \hline 63.5 \text{ } C_{\max} \\ -0.5 \\ \hline 62.5 \text{ } C_{\min} \end{array} \quad 1 \div 2 = 0.5$$

$$A_{\min} = \frac{1}{2} a_{\min} b_{\min} C_{\min}$$

$$A_{\min} = \frac{1}{2} (7.75)(5.15) \sin(62.5) \dots \text{cm}^2$$

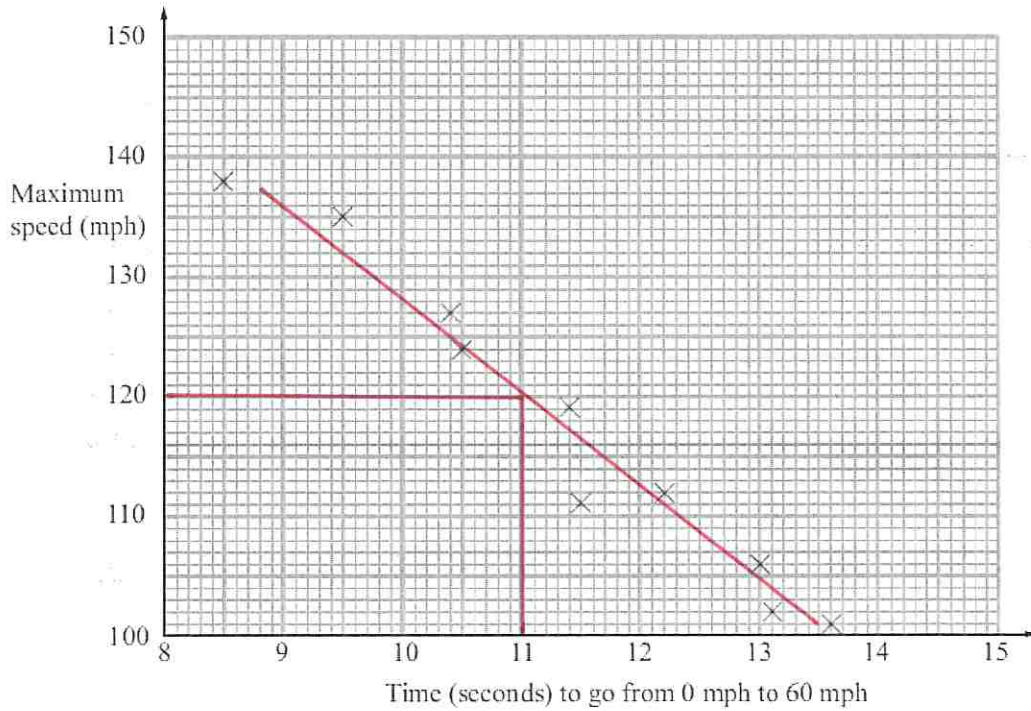
(Total 3 marks)

sin graphs tells us
 $\sin(62.5)$ is less than
 $\sin(63.5)$ \therefore choose 62.5

$$= \underline{\underline{17.70140994 \text{ cm}^2}}$$

Scatter Graphs

8. The scatter graph shows some information about 10 cars. It shows the time, in seconds, it takes each car to go from 0 mph to 60 mph. For each car, it also shows the maximum speed, in mph.



- (a) What type of correlation does this scatter graph show?

Negative

(1)

The time a car takes to go from 0 mph to 60 mph is 11 seconds.

- (b) Estimate the maximum speed for this car.

120

..... mph

(2)

(Total 3 marks)

9. Alex and Ben go to a cafe with some friends.

Alex buys 4 cups of coffee and 3 cups of tea.
He pays a total of £6.95

Ben buys 5 cups of coffee and 2 cups of tea.
He pays a total of £7.20

Work out the cost of each cup of coffee and the cost of each cup of tea.

Alex

$$4c + 3t = 6.95 \quad (1)$$

Ben

$$5c + 2t = 7.20 \quad (2)$$

(1) $\times 2$:

$$8c + 6t = 13.90 \quad (3)$$

(2) $\times 3$:

$$15c + 6t = 21.60 \quad (4)$$

(4) $-$ (3)

$$\begin{array}{r} 15c + 6t = 21.60 \\ 8c + 6t = 13.90 \\ \hline 7c = 7.70 \end{array} \quad (5)$$

$$7c = 7.70$$

$$c = 1.10$$

$c = 1.10$ into (2)

$$5c + 2t = 7.20$$

$$5(1.10) + 2t = 7.20$$

$$5.50 + 2t = 7.20$$

$$2t = 1.70$$

$$t = 0.85$$

Same Terms Opposite Plus

(-5.50)
($\div 2$)

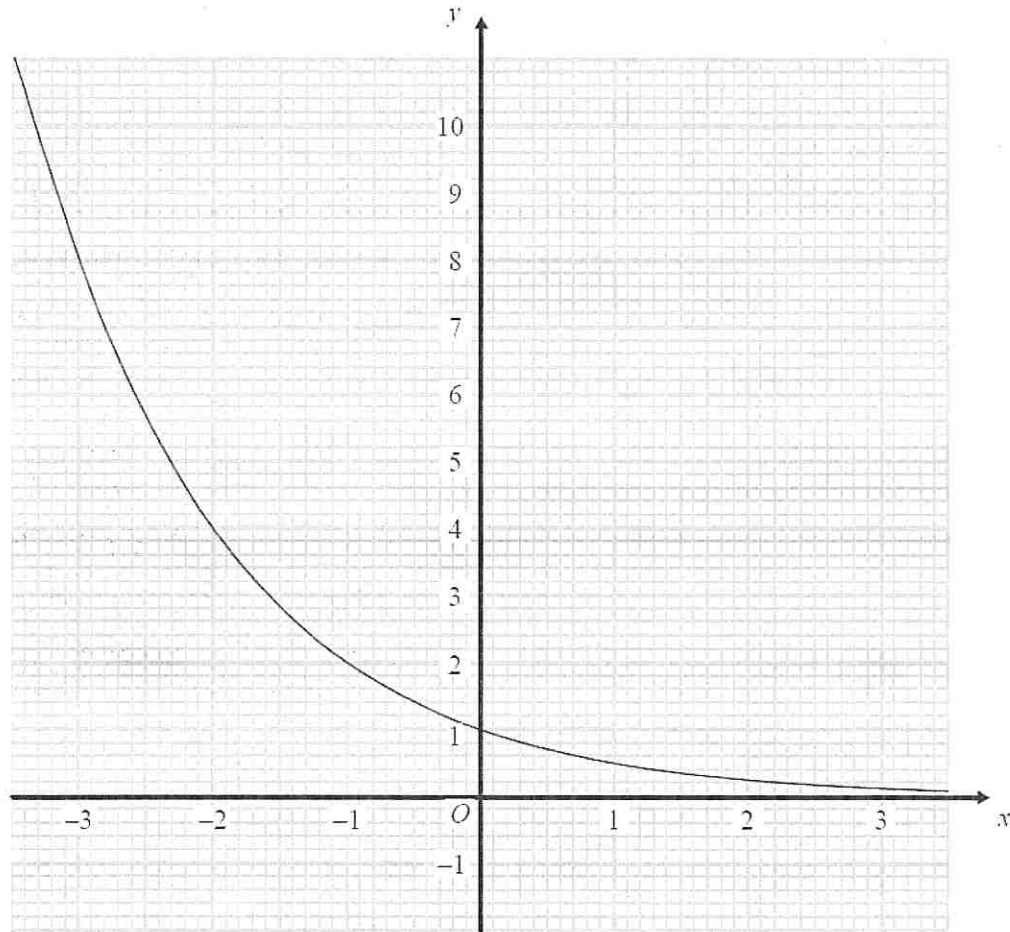
Cup of coffee..... £1.10

Cup of tea..... 85p

(Total 5 marks)

Exponential Graphs

10.



The graph of $y = k^x$, where k is a positive constant, is shown above.

Find the value of k .

Pick a point! NOTE: (0,1) isn't helpful since anything to the power 0 = 1

∴ let's pick (-2, 4)

∴
$$\begin{array}{l} 4 = k^{-2} \\ 2 = k^{-1} \\ \frac{1}{2} = k \end{array}$$

ANS

$k = \frac{1}{2}$

(Total 2 marks)

Converting Units (currency)

11. In the USA, Sam pays 20.88 US Dollars for 6 US gallons of petrol.
In Russia, Leon pays 800 Roubles for 25.58 litres of petrol.

Use the information in the table to compare the prices of petrol in the two countries.

- 1 US gallon = 3.79 litres
- 1 Euro = 40.63 Roubles
- 1 US Dollar = 0.77 Euros

Put everything into litres and euros

USA

$$6G = \$20.88$$

$$1G = 3.79L \quad | \quad 22.74L = \$20.88$$

$$\$1 = €0.77 \quad | \quad 22.74L = \cancel{€}16.0716 \downarrow \times 0.77$$

$$WANTIL \quad | \quad 1L = €0.7070\dots$$

(=22.74)

Russia

$$25.58L = 800R$$

$$€1.4063R \quad | \quad 25.58L = €19.68\dots \downarrow \div 40.63$$

$$WANTIL \quad | \quad 1L = €0.7897\dots$$

(=25.58)

Conclusion: Petrol is better value for money in the USA.

(Total 5 marks)

Area of a trapezium

13. A trapezium $ABCD$ has an area of $5\sqrt{6} \text{ cm}^2$.

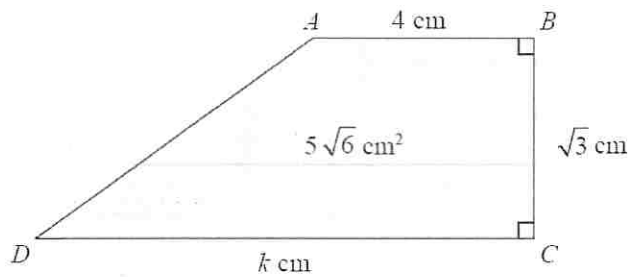


Diagram NOT accurately drawn

$$\begin{aligned} AB &= 4 \text{ cm.} \\ BC &= \sqrt{3} \text{ cm.} \\ DC &= k \text{ cm.} \end{aligned}$$

Calculate the value of k , giving your answer in the form $a\sqrt{b} - c$, where a , b and c are positive integers. Show each step in your working.

$$\begin{array}{l|l} A = \frac{h(a+b)}{2} & 5\sqrt{6} = \frac{\sqrt{3}(4+k)}{2} \\ (\times 2) & 10\sqrt{6} = \sqrt{3}(4+k) \\ (\div \sqrt{3}) & 10\sqrt{2} = 4+k \\ (- 4) & \underline{\underline{10\sqrt{2} - 4 = k}} \end{array}$$

$$k = \underline{\underline{10\sqrt{2} - 4}}$$

(Total 3 marks)

Volume of cylinders

14. The diagram shows a large tin of pet food in the shape of a cylinder.

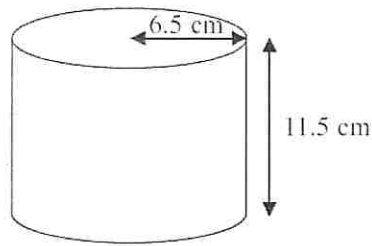


Diagram NOT accurately drawn

The large tin has a radius of 6.5 cm and a height of 11.5 cm.

A pet food company wants to make a new size of tin.

The new tin will have a radius of 5.8 cm.
It will have the same volume as the large tin.

Calculate the height of the new tin.
Give your answer correct to one decimal place.

CSA of a cylinder is a circle!
↓
FRONT FACE

$$V = CSA \times h$$

$$V = \pi r^2 \times h$$

NEW TIN

$$V = \pi r^2 \times h$$

(volume is the same)

$$(\div \pi)$$

$$(\div (5.8)^2)$$

CSA is a circle

$$V = \pi (6.5)^2 \times 11.5$$

$$V = \frac{3887}{8} \pi$$

$$\frac{3887\pi}{8} = \pi (5.8)^2 \times h$$

$$\frac{3887}{8} = (5.8)^2 h$$

$$14.443... = h$$

$$14.4 = h$$

..... 14.4 cm
(Total 3 marks)

Expanding Brackets and Simplifying Algebraic Fractions

15. Prove that, for all positive values of n ,

$$\frac{(n+2)^2 - (n+1)^2}{2n^2 + 3n} = \frac{1}{n}$$

Numerator:

expand

collect

expand

collect

Denominator:

$\frac{\text{Numerator}}{\text{Denominator}}$

Simplify

$$\begin{aligned} &= (n+2)^2 - (n+1)^2 = (n+2)(n+2) - [(n+1)(n+1)] \\ &= n^2 + 2n + 2n + 4 - [n^2 + n + n + 1] \\ &= n^2 + 4n + 4 - [n^2 + 2n + 1] \\ &= n^2 + 4n + 4 - n^2 - 2n - 1 \\ &= 2n + 3 \end{aligned}$$

$$= 2n^2 + 3n = n(2n + 3)$$

$$= \frac{2n+3}{n(2n+3)}$$

$$= \frac{1}{n} \quad \square$$

(Total 4 marks)

16. Make r the subject of the formula $p = \frac{2r+5}{r-3}$

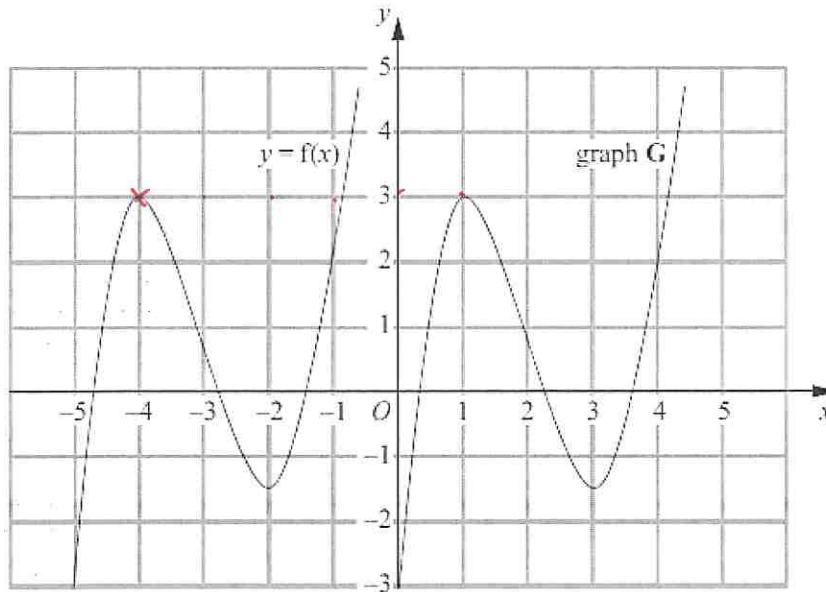
Rearranging Complex Formula

$$\begin{aligned} &P = \frac{2r+5}{r-3} \\ \text{(x(r-3))} & \quad P(r-3) = 2r+5 \\ \text{expand} & \quad pr - 3p = 2r+5 \\ \text{(+3p)} & \quad pr = 2r+5+3p \\ \text{(-2r)} & \quad pr - 2r = 5+3p \\ \text{factorise} & \quad r(p-2) = 5+3p \\ \text{(÷(p-2))} & \quad r = \frac{5+3p}{p-2} \end{aligned}$$

$$r = \frac{5+3p}{p-2}$$

(Total 4 marks)

17. The graph of $y = f(x)$ is shown on the grid.



The graph **G** is a translation of the graph of $y = f(x)$.

- (a) Write down, in terms of f , the equation of graph **G**.

Same as $f(x)$ but 5 to the right = $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$
 $f(x-a)$ is a translation of $\begin{pmatrix} a \\ 0 \end{pmatrix}$

$y = f(x-5)$ (1)

The graph of $y = f(x)$ has a maximum point at $(-4, 3)$.

- (b) Write down the coordinates of the maximum point of the graph of $y = f(-x)$.

$f(-x)$
 ↑
 Inputs have been negated
 $\therefore (-4, 3)$ changes to $(4, 3)$

(.....,)
 (2)

(Total 3 marks)

Velocity Time Graphs

18. A parachutist jumps out of a plane.

This graph shows information about the velocity, v m/s, of the parachutist t seconds after he jumped.

- (a) Work out an estimate for the acceleration of the parachutist when $t = 8$

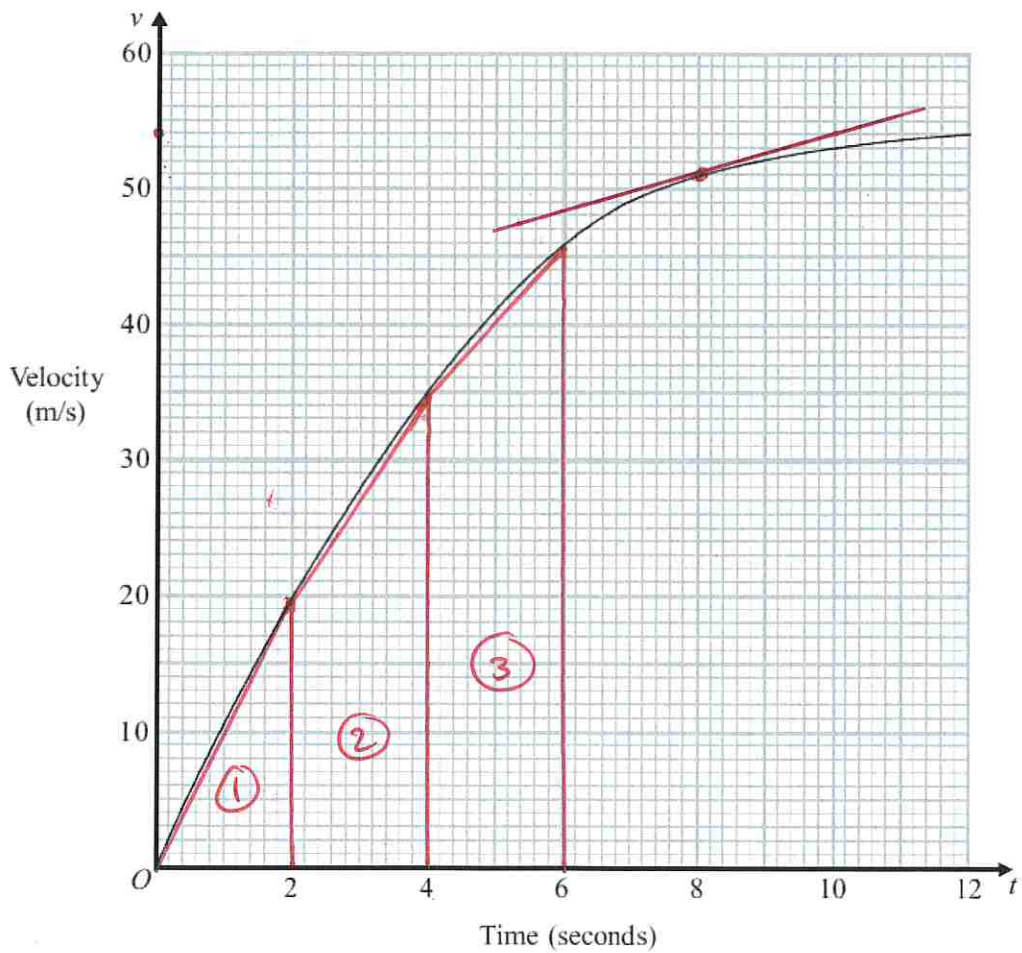
Draw tangent

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$(8, 51)$ and $(10, 54)$

$$m = \frac{54 - 51}{10 - 8} = \frac{3}{2}$$

..... 1.5 m/s²
(3)



Area under the graph

(b) Work out an estimate for the distance the parachutist falls in the first 6 seconds.

① $A = \frac{b \times h}{2}$	$A = \frac{2 \times 20}{2} = 20 \text{ metres}$
② $A = \frac{h(a+b)}{2}$	$A = \frac{2(20+35)}{2} = 55 \text{ metres}$
③ $A = \frac{h(a+b)}{2}$	$A = \frac{2(35+46)}{2} = 81 \text{ metres}$
Total Distance	$20\text{m} + 55\text{m} + 81\text{m} = \underline{156\text{m}}$

..... 156 m

(3)

(Total 6 marks)

19. S is inversely proportional to the cube of t .

Inverse Proportion

When $t = 4$, $S = \frac{1}{2}$

Find the value of S when $t = 8$

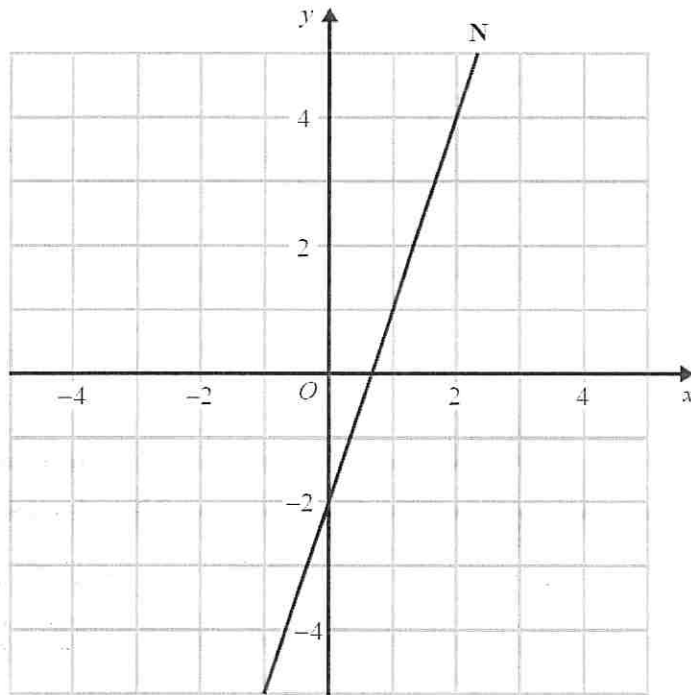
Inverse Proportion	$S \propto \frac{1}{t^3}$		
	$S = \frac{k}{t^3}$	→	$k = 32$
$S = 0.5, t = 4$	$0.5 = \frac{k}{(4)^3}$	↘	$t = 8$
	$0.5 = \frac{k}{64}$		
(x64)	$32 = k$		$S = \frac{32}{t^3}$
			$S = \frac{32}{(8)^3}$
			$S = \frac{32}{512} = \underline{\underline{\frac{1}{16}}}$

..... $\frac{1}{16}$

(Total 4 marks)

Perpendicular Lines

20. The line N is drawn below.



Find an equation of the line perpendicular to line N that passes through the point (0, 1).

$y = mx + c$
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
 $m_1 \times m_2 = -1$
 $y = -\frac{1}{3}x + c$
 at (0, 1)
 $y = mx + c$

For gradient of perpendicular we need gradient of N first
 (1, 1) and (2, 4) : $\frac{4-1}{2-1} = \frac{3}{1} = 3$
 \therefore gradient of perpendicular = $-\frac{1}{3}$
 $1 = -\frac{1}{3}(0) + c$
 $1 = c$
 $y = -\frac{1}{3}x + 1$

$y = -\frac{1}{3}x + 1$
 (Total 3 marks)

Gradients / Vectors / Straight Lines

21. The points A , B and C lie in order on a straight line.

The coordinates of A are $(2, 5)$

The coordinates of B are $(4, p)$

The coordinates of C are $(q, 17)$

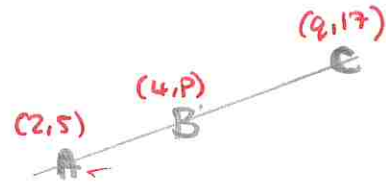
Given that $AC = 4AB$, find the values of p and q .

$$\vec{AC} = \begin{pmatrix} q-2 \\ 12 \end{pmatrix}$$

$$\therefore \vec{AB} = \begin{pmatrix} \frac{q-2}{4} \\ 3 \end{pmatrix}$$

$$5+3=8$$

$$\therefore \text{Coordinates of } B = (4, \underline{8}) \quad \therefore p = \underline{8}$$



Also gradient is the same all the way up (since straight line)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-5}{4-2} = \frac{3}{2} \leftarrow \text{use to find } q.$$

$(2, 5) (4, 8)$

$$\frac{3}{2} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{17-5}{q-2} = \frac{12}{q-2}$$

$p = \dots\dots\dots 8$
 $q = \dots\dots\dots 10$

(Total 3 marks)

$(2, 5) (q, 17)$

TOTAL FOR PAPER IS 80 MARKS

$$\therefore \frac{3}{2} = \frac{12}{q-2}$$

cross multiply	$3(q-2) = 24$
$(\div 3)$	$q-2 = 8$
$(+2)$	$q = \underline{10}$