

GCSE Mathematics

Practice Tests: Set 4

Paper 3H (Calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator.

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**



Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. Here is a list of ingredients needed to make 20 cookies.

Cookies	
Ingredients to make 20 cookies.	
250 g butter	
120 g caster sugar	
300 g flour	

Sam is going to make some cookies.

She has these ingredients.

625 g butter

360 g caster sugar

1000 g flour

Work out the greatest number of cookies that Sam can make with her ingredients.
You must show your working.

Batches using butter	$625 \div 250 = 2\frac{1}{2}$ batches
Batches using sugar	$360 \div 120 = 3$ batches
Batches using flour	$1000 \div 300 = 3\frac{1}{3}$ batches
Cookies using butter	$2\frac{1}{2} \times 20 = 50$
Cookies using sugar	$3 \times 20 = 60$
Cookies using flour	$3\frac{1}{3} \times 20 \approx 66$
Conclusion	$\therefore 50$ is the maximum

50

(Total 3 marks)

2. Celina and Zoe both sing in a band.

One evening the band plays for 80 minutes.
Celina sings for 65% of the 80 minutes.

Zoe sings for $\frac{5}{8}$ of the 80 minutes.

Celina sings for more minutes than Zoe sings.

Work out for how many more minutes.
You must show all your working.

<u>Celina:</u>	$65\% \text{ of } 80 = 0.65 \times 80$ $= 52 \text{ mins}$
<u>Zoe:</u>	$\frac{5}{8} \text{ of } 80 = 50 \text{ mins}$
Difference	$52 - 50 = \underline{\underline{2 \text{ mins}}}$

.....2.....minutes
(Total 4 marks)

3.

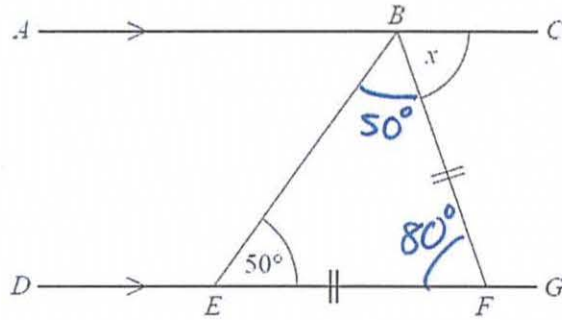


Diagram NOT
accurately drawn

ABC is a straight line.
 $DEFG$ is a straight line.
 AC is parallel to DG .
 $EF = BF$.
 Angle $BEF = 50^\circ$.

Work out the size of the angle marked x .
 Give reasons for your answer.

$$\hat{BEF} = \hat{EBF} = 50^\circ$$

$$\hat{BFE} = 180^\circ - 50^\circ - 50^\circ = 80^\circ$$

$$\hat{BFE} = x^\circ = 80^\circ$$

Base angles of isosceles triangle the same

180° in a triangle

Alternate angles in parallel lines the same.

80°

(Total 4 marks)

4. (a) Simplify $(c^2k^5)^4$

$$= (c^2k^5)^4 = (c^2k^5)(c^2k^5)(c^2k^5)(c^2k^5)$$

$$= \underline{\underline{c^8k^{20}}}$$

$$c^8k^{20}$$

(1)

- (b) Expand and simplify $(3x + 5)(4x - 1)$

expand	$(3x+5)(4x-1)$ $12x^2 - 3x + 20x - 5$ $12x^2 + 17x - 5$
collect	

$$12x^2 + 17x - 5$$

(2)

- (c) Solve $x^2 - 3x - 10 = 0$

Solving Quadratics

factorise	$(x-5)(x+2) = 0$ $x = \underline{\underline{5}} \text{ or } x = -2$
solve	

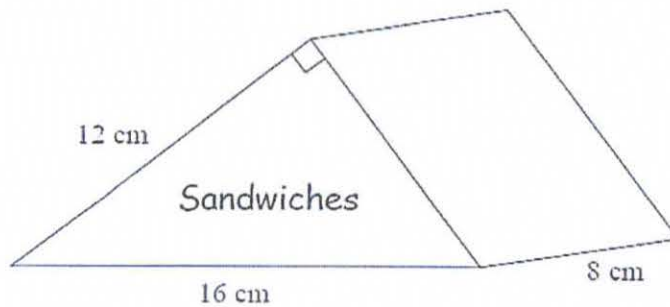
$$x = \underline{\underline{5 \text{ or } -2}}$$

(3)

(Total 6 marks)

Pythagoras and Volume of Prisms

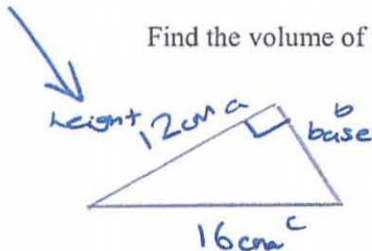
5. Here is a sandwich packet.



The packet is in the shape of a triangular prism.
The triangular faces are right-angled triangles.

Find the volume of the sandwich packet.

Cross-section "front face"



$$\text{Volume} = \text{CSA} \times \text{length}$$

$$\text{CSA} = \frac{b \times h}{2}$$

$$\text{Volume} = \text{CSA} \times \text{length}$$

NEED THE BASE:

$$\text{Pythagoras} : a^2 + b^2 = c^2$$

$$12^2 + b^2 = 16^2$$

$$144 + b^2 = 256$$

$$(-144)$$

$$\sqrt{\text{ANS}}$$

$$b^2 = 112$$

$$b = \sqrt{112} = 10.583...$$

$$\text{CSA} = \frac{12 \times 10.583...}{2}$$

$$= 63.4980... = \text{ANS}$$

$$V = \text{ANS} \times 8$$

$$= 507.9842517 = 508_{\text{cm}^3} \text{ (3 s.f.)}$$

$$508_{\text{cm}^3}$$

(Total 5 marks)

Rearranging Formula

6. Make p the subject of the formula $y = 3p^2 - 4$.

$$\begin{array}{l|l} & y = 3p^2 - 4 \\ (+4) & y + 4 = 3p^2 \\ (\div 3) & \frac{y+4}{3} = p^2 \\ \sqrt{\text{ANS}} & \pm \sqrt{\frac{y+4}{3}} = p \end{array}$$

$$p = \pm \sqrt{\frac{y+4}{3}}$$

(Total 3 marks)

7. There are 10 boys and 20 girls in Mrs Brook's class. 30 children
Mrs Brook gave all the class a test.

The mean mark for all the class is 60.

The mean mark for the girls is 56.

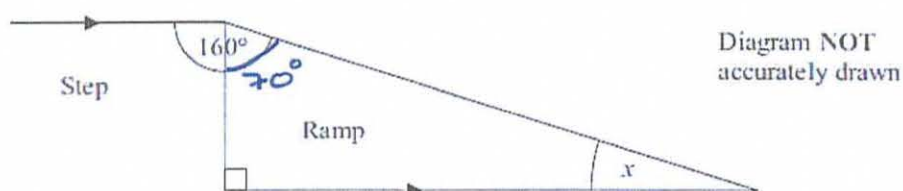
Work out the mean mark for the boys.

Total marks	$30 \times 60 = 1800$
Total girl marks	$20 \times 56 = 1120$
Total boy marks	$1800 - 1120 = 680$
mean boy mark	$680 \div 10 = \underline{\underline{68}}$

68

(Total 3 marks)

8. The diagram shows a ramp next to a step.



- (i) Work out the size of the angle marked x .

$$\begin{array}{l|l} \text{Right angle} & 160 - 90 = 70^\circ \\ 180^\circ \text{ in a triangle} & 180 - 70^\circ - 90^\circ = 20^\circ \end{array}$$

$$x = \dots\dots\dots 20^\circ$$

- (ii) Give a reason for your answer.

180° in a triangle

.....

.....

.....

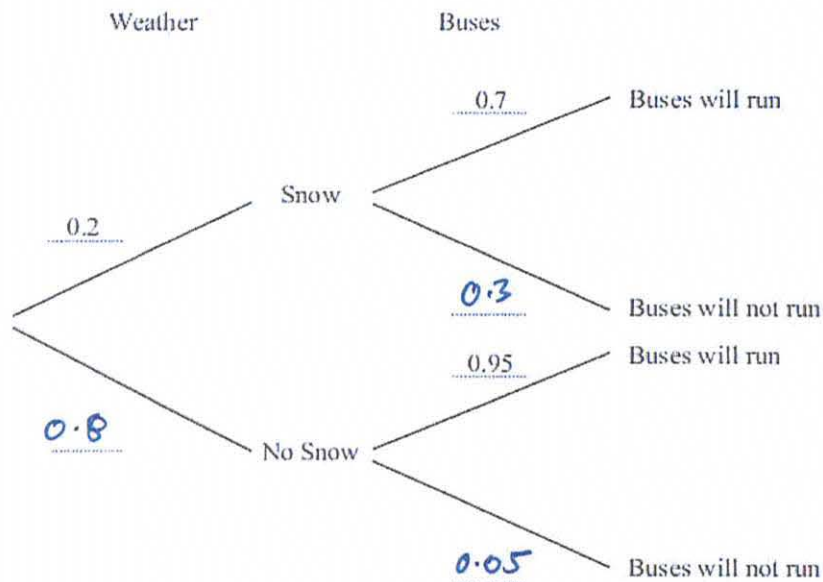
.....

(Total 3 marks)

Dependent Probability Trees

9. The decision tree diagram gives information about the probability of snow for the first 50 days in winter and the probability of whether buses will run or not run.

(a) Complete the decision tree diagram.



(2)

- (b) Work out the probability that it will snow and the buses will not run.

$$P(\text{Snow, Not Run}) = 0.2 \times 0.3$$

$$= 0.06$$

0.06

(2)

(Total 4 marks)

10. A baker makes jam rolls.

The baker uses flour, butter and jam in the ratio 8 : 4 : 5 to make jam rolls.

The table shows the cost per kilogram of some of these ingredients.

Cost per kilogram	
Flour	40p
Butter	£2.50
Jam	£1.00

The total weight of the flour, butter and jam for each jam roll is 425 g.

Work out the cost of these ingredients for 200 jam rolls.

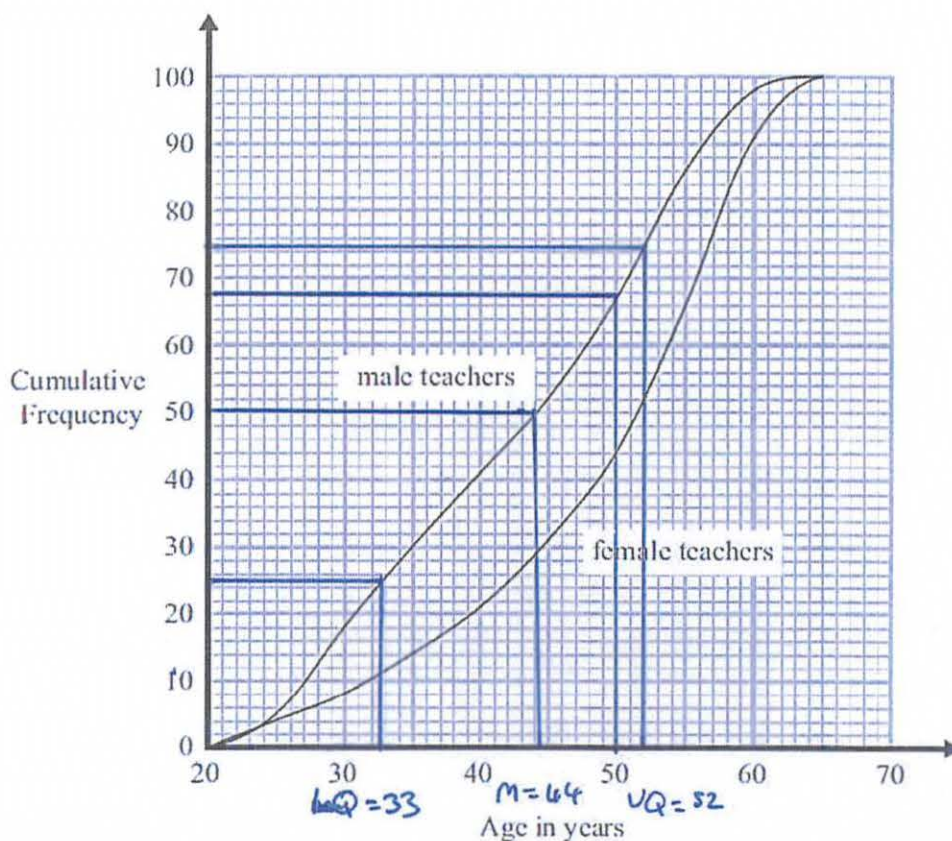
Ratio F:B:J	
8:4:5	$8+4+5 = 17 \text{ parts}$
	$17 \text{ parts} = 425 \text{ g}$
($\div 17$)	$1 \text{ part} = 25 \text{ g}$
F:B:J	8:4:5
($\times 25$)	200:100:125 per jam roll (grams)
200 rolls ($\times 200$)	40000:20000:25000 (kg) (Needed though...)
($\div 1000$)	40:20:25 (kg)
Cost of Flour	$40 \times 40\text{p} = 1600\text{p} = \pounds 16$
Cost of Butter	$\pounds 2.50 \times 20 = \pounds 50$
Cost of Jam	$\pounds 1 \times 25 = \pounds 25$
Total Cost	$\pounds 16 + \pounds 50 + \pounds 25 = \underline{\underline{\pounds 91}}$

£ 91

(Total 6 marks)

11. A student did a survey of teachers' ages.

The cumulative frequency curves show information about the ages of the male teachers and the ages of the female teachers.



- (a) Find an estimate for the number of the male teachers older than 50 years of age.

68 teachers are up until the age of 50

100 teachers in total

$$100 - 68 = \underline{\underline{32}}$$

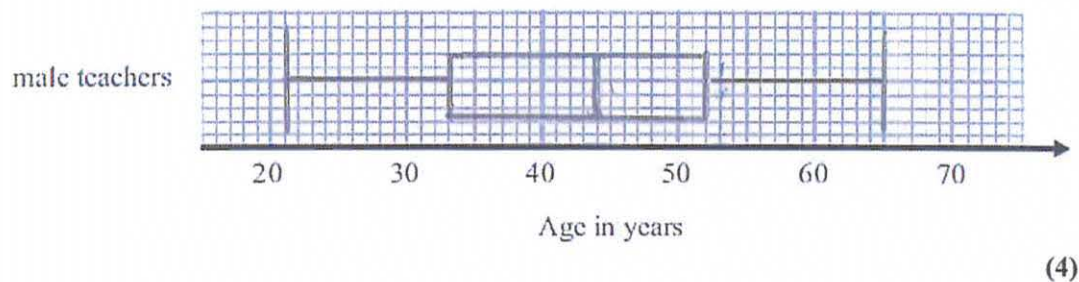
32

(2)

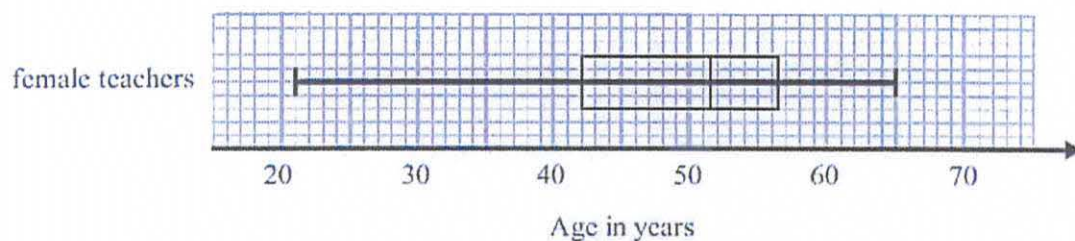
Box Plots

The youngest male teacher was 21 years of age.
The oldest male teacher was 65 years of age.

- (b) On the grid, draw a box plot to show the distribution of the ages of the male teachers.



The box plot below shows information about the ages of the female teachers.



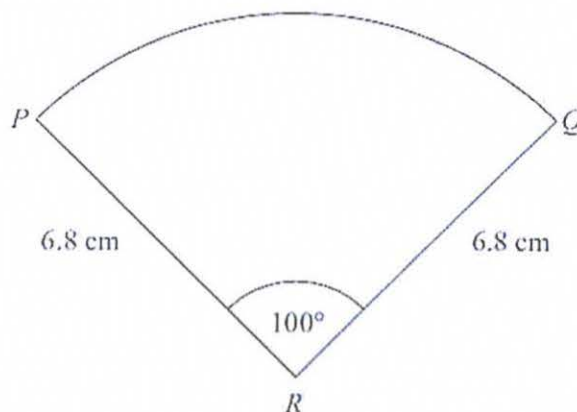
- (c) Make two comparisons between the distributions of the ages of male teachers and female teachers.

The average male teacher is younger than the average female teacher (lower median). The spread of female teachers ages is less than the male ages (smaller IQR).

(2)

(Total 8 marks)

12. The diagram shows a sector of a circle of radius 6.8 cm.



PQ is an arc of the sector.
Angle $PRQ = 100^\circ$.

Work out the perimeter of the sector.
Give your answer correct to 3 significant figures.

$PQ = \text{Arc Length}$

$$\text{Arc} = \frac{\theta}{360} \times \pi d \quad (d=13.6)$$

$$= \frac{100}{360} \times \pi (13.6) = 11.86823891 = \text{ANS}$$

Perimeter = $2r + \text{Arc}$

$$= 6.8 + 6.8 + \text{ANS}$$

$$= 25.5 \text{ cm (3 s.f.)}$$

25.5

..... cm

(Total 3 marks)

13. f is the function $f(x) = 2x + 5$.

(a) Find $f(3)$.

$$f(3) = 2(3) + 5 \\ = 6 + 5 = 11$$

11

(1)

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) =$

$$\begin{array}{l|l} & y = 2x + 5 \\ (-5) & y - 5 = 2x \\ (\div 2) & \frac{y-5}{2} = x \\ \text{Notation} & \frac{x-5}{2} = f^{-1}(x) \end{array}$$

$$f^{-1}(x) = \frac{x-5}{2}$$

(2)

g is the function $g(x) = x^2 - 25$.

(c) Find $g(-3)$.

$$g(-3) = (-3)^2 - 25 \\ = 9 - 25 \\ = -16$$

-16

(1)

(d) (i) Find $gf(x)$.

Give your answer as simply as possible.

$$f(x) = 2x + 5 \quad g(x) = x^2 - 25$$

Expand
collect
factorize

$$\begin{aligned} gf(x) &= (2x+5)^2 - 25 \\ &= (2x+5)(2x+5) - 25 \\ &= 4x^2 + 10x + 10x + 25 - 25 \\ &= 4x^2 + 20x \\ &= 4x(x+5) \\ gf(x) &= 4x(x+5) \end{aligned}$$

(ii) Solve $gf(x) = 0$.

$$gf(x) = 4x(x+5)$$

Solve

$$4x(x+5) = 0$$

$$x = 0 \quad \underline{\text{OR}} \quad x = -5$$

$$x = 0 \text{ or } -5$$

(5)

(Total 9 marks)

14. Dan does an experiment to find the value of π .
He measures the circumference and the diameter of a circle.

He measures the circumference, C , as 170 mm to the nearest millimetre.
He measures the diameter, d , as 54 mm to the nearest millimetre.

Dan uses $\pi = \frac{C}{d}$ to find the value of π .

Calculate the upper bound and the lower bound for Dan's value of π .

C Error

$$\begin{array}{r} 170 \\ +0.5 \\ \hline 170.5 \text{ } C_{\text{max}} \\ -0.5 \\ \hline 169.5 \text{ } C_{\text{min}} \end{array} \quad 1 \div 2 = 0.5$$

d Error

$$\begin{array}{r} 54 \\ +0.5 \\ \hline 54.5 \text{ } d_{\text{max}} \\ -0.5 \\ \hline 53.5 \text{ } d_{\text{min}} \end{array} \quad 1 \div 2 = 0.5$$

$$\pi_{\text{max}} = \frac{C_{\text{max}}}{d_{\text{min}}}$$

$$\pi_{\text{max}} = \frac{170.5}{53.5} = \underline{\underline{3.19...}} \text{ (3 s.f.)}$$

$$\pi_{\text{min}} = \frac{C_{\text{min}}}{d_{\text{max}}}$$

$$\pi_{\text{min}} = \frac{169.5}{54.5} = \underline{\underline{3.11...}} \text{ (3 s.f.)}$$

upper bound = $\underline{\underline{3.19}}$

lower bound = $\underline{\underline{3.11}}$

(Total 4 marks)

Adding Algebraic Fractions (combining)

15. Simplify

$$\frac{x+1}{2} + \frac{x+3}{3}$$

cross multiply

$$\frac{3(x+1) + 2(x+3)}{6}$$

expand

$$\frac{3x+3+2x+6}{6}$$

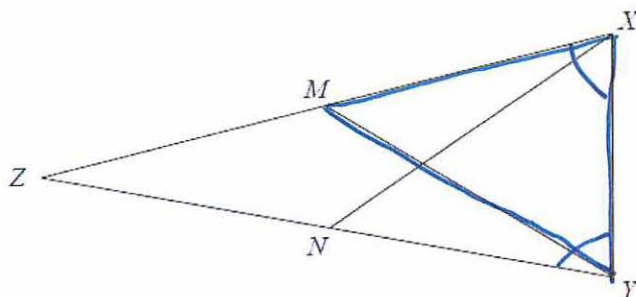
Collect

$$\frac{5x+9}{6}$$

$$\frac{5x+9}{6}$$

(Total 3 marks)

16. The diagram shows an isosceles triangle XYZ .



$$XZ = YZ$$

M is the midpoint of XZ .
 N is the midpoint of YZ .

Prove that triangle XMN is congruent to triangle YNM .
 Give reasons for each stage of your working.

⑤ $XZ = YZ$

$$XZ = YZ$$

$$\therefore \frac{1}{2}XZ = \frac{1}{2}YZ$$

⑤ $\therefore XM = YN$

⑥ $\angle XZM = \angle YZN$

By SAS, $\triangle XMN$ and $\triangle YNM$ are congruent.



Shared side

midpoint

Base angles of isosceles triangle are the same.

Conclusion

(Total 4 marks)

17. $A : B : C = x + 1 : 3 : x - 1$

$$A + B + C = 60$$

$$B = 30$$

Work out the value of x .

	A	:	B	:	C	Total
	$x+1$:	3	:	$x-1$	$2x+3$
Given $B = 30$ (Ratio has been $\times 10$)	$10(x+1)$:	30	:	$10(x-1)$	$10(x+1) + 30 + 10(x-1)$
Equate totals...	$A + B + C = 10(x+1) + 30 + 10(x-1) = 60$					
$A + B + C = 60$						
Expand	$10x + 10 + 30 + 10x - 10 = 60$					
collect	$20x + 30 = 60$					
(-30)	$20x = 30$					
$(\div 20)$	$x = 1.5$					

1.5

(Total 5 marks)

18. Prove that

$$(2n+3)^2 - (2n-3)^2 \text{ is a multiple of 8}$$

for all positive integer values of n .

Expand	$(2n+3)(2n+3) - (2n-3)(2n-3)$
...	$= 4n^2 + 6n + 6n + 9 - [4n^2 - 6n - 6n + 9]$
	$= 4n^2 + 6n + 6n + 9 - 4n^2 + 6n + 6n - 9$
Collect	$= 24n$
	$= 8 \times 3n$
Conclusion	$3n$ is multiplied by 8 $\therefore 24n$ is a multiple of 8. \square

(Total 3 marks)

TOTAL FOR PAPER IS 80 MARKS