

# GCSE Mathematics

## Practice Tests: Set 4

### Paper 1H (Non-calculator)

**Time: 1 hour 30 minutes**

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator.

#### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**



#### Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

#### Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. Write these numbers in order of size.  
Start with the smallest number.

$$2^5 \qquad 64^{\frac{1}{2}} \qquad 4^3 \qquad 8^{\frac{1}{3}} \qquad 16 \qquad 64^0$$

You must show clearly how you got your answer.

$a^{\frac{1}{n}} = \sqrt[n]{a}$	$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$	⑤
	$64^{\frac{1}{2}} = \sqrt{64} = 8$	③
	$4^3 = 4 \times 4 \times 4 = 64$	⑥
$a^{\frac{1}{n}} = \sqrt[n]{a}$	$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$	②
	$16 = 16$	④
$a^0 = 1$	$64^0 = 1$	①

$$64^0, 8^{\frac{1}{3}}, 64^{\frac{1}{2}}, 16, 2^5, 4^3$$

(Total 3 marks)

# Relative Frequency

2. There are 50 counters in a bag.

The counters are blue or yellow or black or white.  
A counter is taken at random from the bag.

The table shows each of the probabilities that the counter will be blue or black or white.

Colour	blue	yellow	black	white
Probability	0.4	0.14	0.3	0.16

Work out the number of yellow counters in the bag.

$$\begin{array}{l|l} \text{Sum of all probabilities} = 1 & P(\text{yellow}) = 1 - 0.4 - 0.3 - 0.16 \\ & = 0.3 - 0.16 \\ & = \underline{\underline{0.14}} \end{array}$$

Relative Frequency

$$\begin{array}{l} 0.14 \times 50 = 1.4 \times 5 \\ = \underline{\underline{7}} \end{array}$$

$$\begin{array}{r} 1.4 \times \\ 5 \\ \hline 7.0 \\ 2 \end{array}$$

7

(Total 4 marks)

3. Buses to Acton leave a bus station every 24 minutes.  
Buses to Barton leave the same bus station every 20 minutes.

A bus to Acton and a bus to Barton both leave the bus station at 9 00 a.m.

When will a bus to Acton and a bus to Barton next leave the bus station at the same time?

Lcm

Acton :	24	48	72	96	120	
Barton :	20	40	60	80	100	120

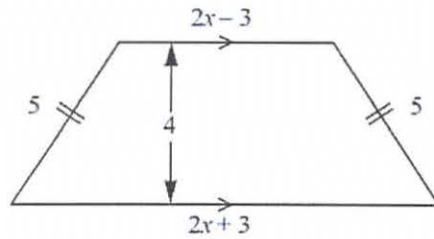
LCM = 120  $\therefore$  leave at the same time  
120 minutes after 9:00 am

9:00 + 2 hours = 11:00 am

11:00 am

(Total 3 marks)

4. Here is a trapezium.



All the measurements are in cm.  
The area of the trapezium is  $18 \text{ cm}^2$ .

Calculate the numerical value of the perimeter of the trapezium.

$$A = \frac{h(a+b)}{2}$$

Given:  $A = 18$

collect

simplify

expand  
( $\div 8$ )

Perimeter:

collect

Sub  $x = \frac{18}{8}$

$$A = \frac{4(2x-3+2x+3)}{2}$$

$$18 = \frac{4(2x-3+2x+3)}{2}$$

$$18 = \frac{4(4x)}{2}$$

$$18 = 2(4x)$$

$$18 = 8x$$

$$\frac{18}{8} = x$$

$$P = 5 + 5 + 2x + 3 + 2x - 3$$

$$P = 10 + 4x$$

$$P = 10 + 4\left(\frac{18}{8}\right)$$

$$= 10 + \frac{72}{8}$$

$$= 10 + 9 = \underline{\underline{19}}$$

19

.....cm

(Total 6 marks)

# Reverse Percentages

5. The normal price of a television is reduced by 30% in a sale.

The sale price of the television is £350

Work out the normal price of the television.

original = 100%	$100\% - 30\% = 70\%$
	£350 = 70%
(÷70)	£5 = 1%
(×100)	<u>£500</u> = 100%

£ 500

(Total 3 marks)

6. Work out an estimate for the value of

$$\frac{6.8 \times 191}{0.051}$$

$$\begin{aligned} 6.8 &\approx 7 \\ 191 &\approx 200 \\ 0.051 &\approx 0.05 \end{aligned}$$

$$\frac{1400}{0.05} = \frac{140000}{5}$$

$$\frac{6.8 \times 191}{0.051} \approx \frac{7 \times 200}{0.05}$$

$$\approx \frac{1400}{0.05}$$

$$\begin{array}{r} 028000 \\ 5 \overline{)140000} \end{array} \approx \underline{\underline{28000}}$$

(Total 3 marks)

7.

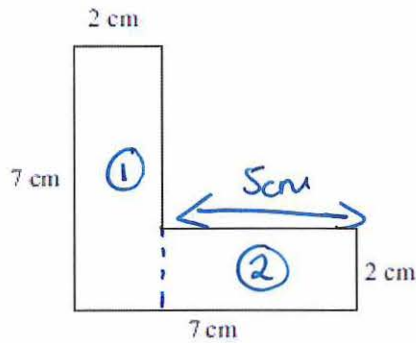
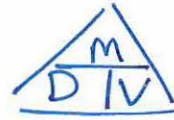


Diagram NOT  
accurately drawn



The diagram shows the cross-section of a solid prism.  
The length of the prism is 2 m. = 200 cm

The prism is made from metal.  
The density of the metal is 8 grams per  $\text{cm}^3$ .

Work out the mass of the prism.

$$m = D \times V$$

$$V = \text{CSA} \times \text{Length}$$

CSA ...

$$V = \text{CSA} \times \text{Length}$$

$$m = D \times V$$

$$\begin{aligned} \text{CSA} &= \text{Area ①} + \text{Area ②} \\ &= (7\text{ cm} \times 2\text{ cm}) + (5\text{ cm} \times 2\text{ cm}) \\ &= 14\text{ cm}^2 + 10\text{ cm}^2 \\ &= 24\text{ cm}^2 \end{aligned}$$

$$\begin{aligned} V &= 24\text{ cm}^2 \times 200\text{ cm} \\ &= 4800\text{ cm}^3 \end{aligned}$$

$$m = 4800\text{ cm}^3 \times 8\text{ g/cm}^3$$

$$m = 38400\text{ g}$$

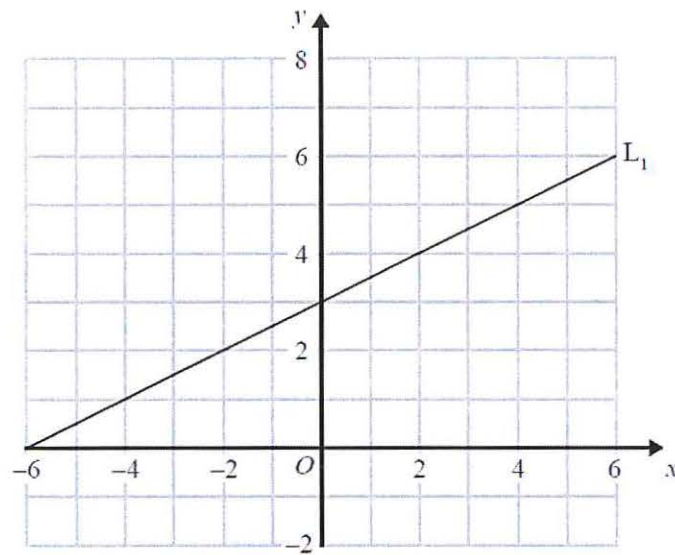
$$\begin{array}{r} 4800 \times 8 \\ \hline 38400 \\ 36 \end{array}$$

38400g

(Total 5 marks)



8. The diagram shows a straight line,  $L_1$ , drawn on a grid.



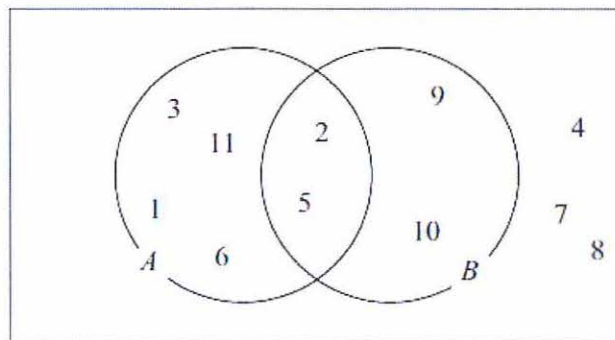
A straight line,  $L_2$ , is parallel to the straight line  $L_1$  and passes through the point  $(0, -5)$ .

Find an equation of the straight line  $L_2$ .

<p><math>L_2</math> is parallel to <math>L_1</math>:</p> $m = \frac{y_2 - y_1}{x_2 - x_1}$ <p><math>(0, 3)</math> and <math>(4, 5)</math></p> $y = \frac{1}{2}x + c$ <p><math>(0, -5)</math></p> $\underline{\underline{y = \frac{1}{2}x - 5}}$	<p><math>L_2</math> has same gradient of <math>L_1</math></p> $m = \frac{5 - 3}{4 - 0} = \frac{2}{4} = \frac{1}{2}$ <p><math>\therefore</math> gradient of <math>L_2 = \frac{1}{2}</math></p> <p>y-intercept = <math>-5</math></p> <hr style="border: 0.5px solid black;"/> <p><math>\underline{\underline{L_2: y = \frac{1}{2}x - 5}}</math></p>
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(Total 3 marks)

9. The Venn diagram shows the numbers 1 to 11



- (a) Work out  $P(A \cup B)$

OR

$$\frac{\text{(Numbers in A or B)}}{\text{Total Numbers}} = \frac{8}{11}$$

$$\frac{8}{11}$$

(2)

- (b) Work out  $P(B')$

NOT B

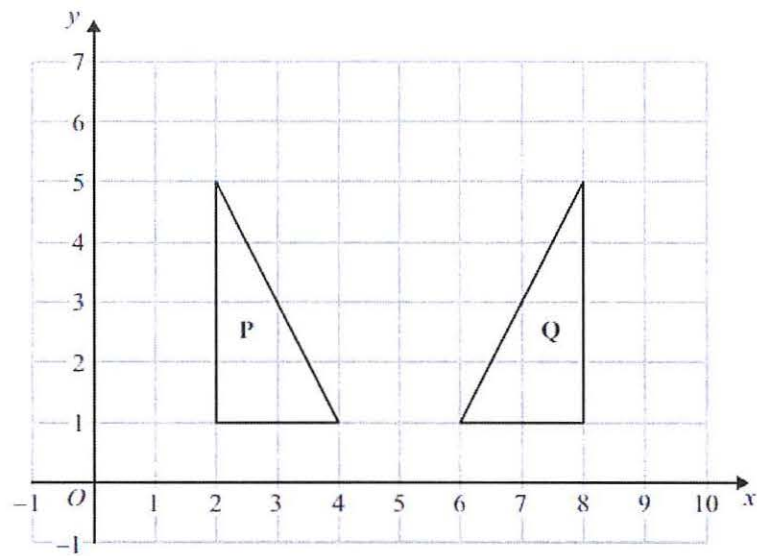
$$\frac{\text{(Numbers NOT in B)}}{\text{Total Numbers}} = \frac{7}{11}$$

$$\frac{7}{11}$$

(2)

(Total 4 marks)

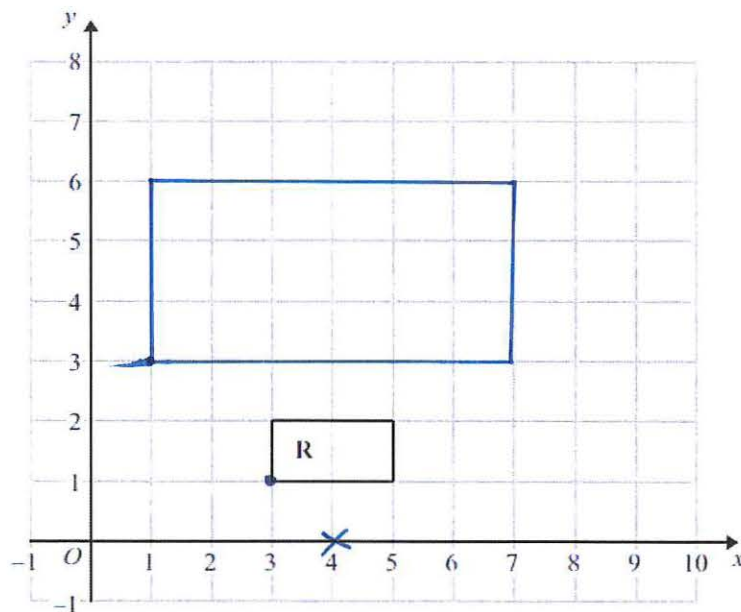
10.



- (a) Describe fully the single transformation that maps triangle **P** onto triangle **Q**.

Reflection in the line  $x = 5$

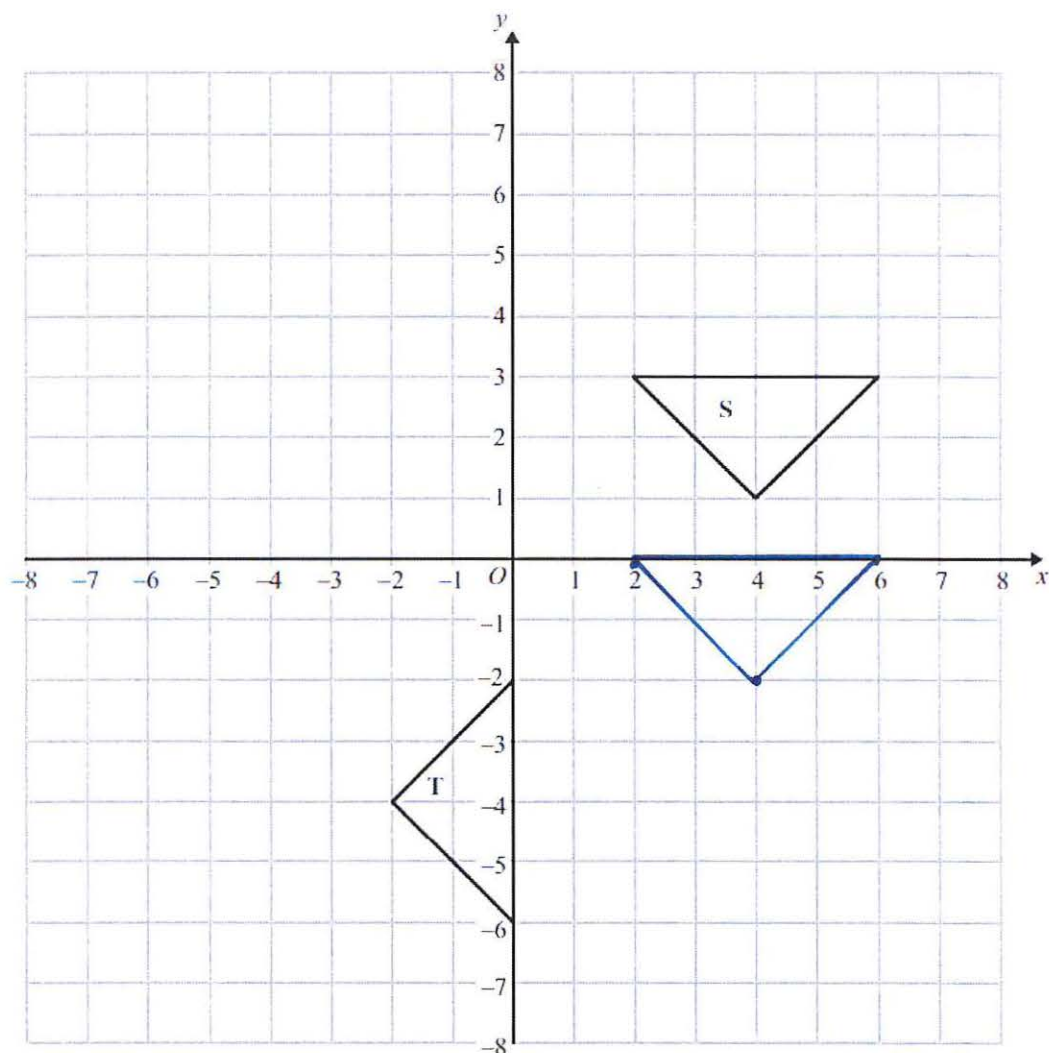
(2)



- (b) Enlarge rectangle **R**, with scale factor 3 and centre (4, 0).

By vectors:  $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$  SF  $\times 3$   $3 \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

(2)



Shape S can be transformed to shape T by the translation  $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$  followed by a rotation.

3 down

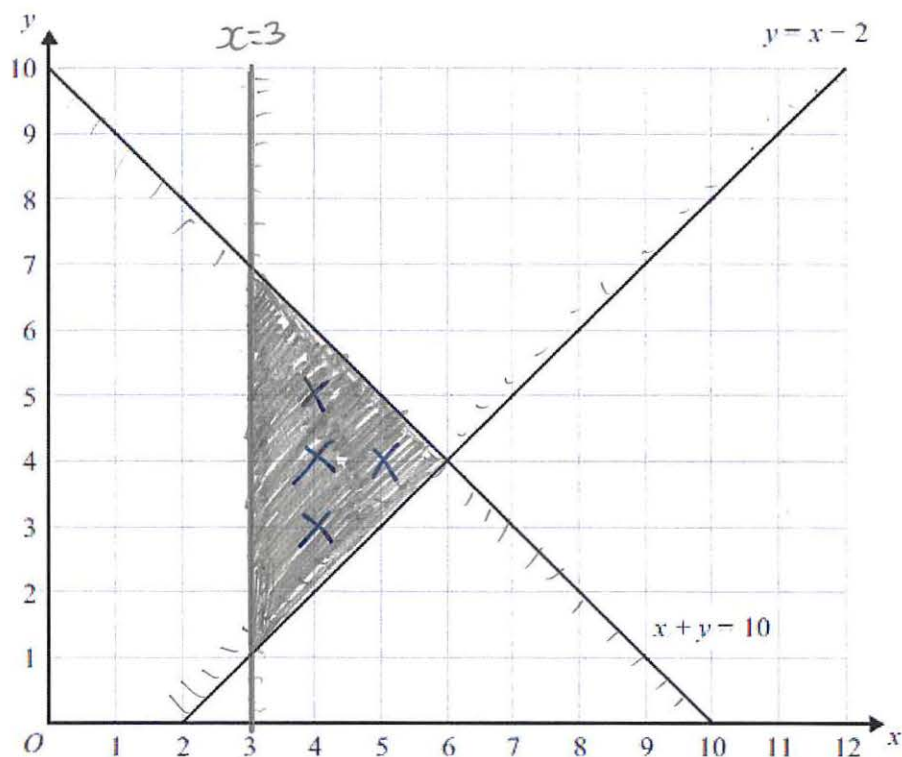
(c) Describe the rotation.

90° clockwise, centre (0, 0)

(3)

(Total 7 marks)

11. The lines  $y = x - 2$  and  $x + y = 10$  are drawn on the grid.



On the grid, mark with a cross (×) each of the points with integer coordinates that are in the region defined by

$(0,0)$  works       $y > x - 2$  Sketched  
 $(0,0)$  works       $x + y < 10$  Sketched.  
 $x > 3$  Needs sketching

Co-ordinates ON THE LINES do not count

(Total 3 marks)

since  $>$  signs, not  $\geq$ .

12. Harry travels from Appleton to Brockley at an average speed of 50 mph.  
He then travels from Brockley to Cantham at an average speed of 70 mph.

Harry takes a total time of 5 hours to travel from Appleton to Cantham.  
The distance from Brockley to Cantham is 210 miles.

Calculate Harry's average speed for the total distance travelled from Appleton to Cantham.

<u>Journey 1</u> (A → B)	<u>Journey 2</u> (B → C)	<u>Total</u> (A → C)
S = 50 mph	S = 70 mph	S = ??? (5)
D = 100 miles (3)	D = 210 miles	D = 310 miles (4)
T = 2 hours (2)	T = 3 hours (1)	T = 5 hours

$$\textcircled{1} \quad T = \frac{D}{S} = \frac{210 \text{ miles}}{70 \text{ mph}} = 3 \text{ hours}$$

$$\textcircled{2} \quad 5 \text{ hours} - 3 \text{ hours} = 2 \text{ hours}$$

$$\textcircled{3} \quad D = S \times T = 50 \text{ mph} \times 2 \text{ hours} = 100 \text{ miles}$$

$$\textcircled{4} \quad 100 \text{ miles} + 210 \text{ miles} = 310 \text{ miles}$$

$$\textcircled{5} \quad S = \frac{D}{T} = \frac{310 \text{ miles}}{5 \text{ hours}}$$

$$= \underline{\underline{62 \text{ mph}}}$$

..... 62 ..... mph  
(Total 4 marks)



13.

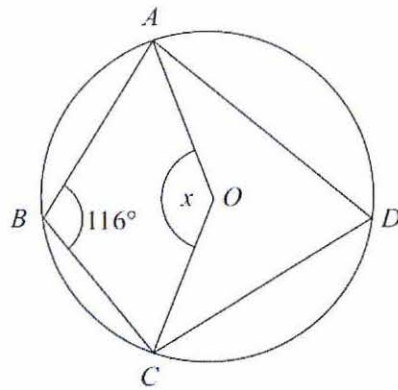


Diagram NOT  
accurately drawn

$A, B, C$  and  $D$  are points on the circumference of a circle with centre  $O$ .  
Angle  $ABC = 116^\circ$ .

Find the size of the angle marked  $x$ .  
Give reasons for your answer.

<p>Reflex angle <math>O = 2 \times 116</math> <math>= 232^\circ</math></p> <p><math>x = 360^\circ - 232^\circ = \underline{\underline{128^\circ}}</math></p>	<p>Angle at the centre is twice that at the circumference</p> <p>Angles around a point <math>= 360^\circ</math></p>
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(Total 4 marks)

14. The  $n$ th term of a quadratic sequence is  $n^2 + 3n - 2$

(a) Find the fourth term of this sequence.

$$\begin{array}{l|l} \text{let } n=4 & n^2 + 3n - 2 \\ & (4)^2 + 3(4) - 2 \\ & 16 + 12 - 2 = \underline{\underline{26}} \end{array} \quad \begin{array}{c} \text{.....} \\ 26 \\ \text{.....} \end{array} \quad (2)$$

Here are the first five terms of a different quadratic sequence.

1                      7                      17                      31                      49

(b) Find, in terms of  $n$ , an expression for the  $n$ th term of this sequence.

$$\begin{array}{ccccccc} & \textcircled{1} & +4 & \rightarrow & +4 & \rightarrow & +4 & \rightarrow \\ & \textcircled{2} & +6 & & +10 & & +14 & & +18 \\ \textcircled{3} & 1 & \rightarrow & 7 & \rightarrow & 17 & \rightarrow & 31 & \rightarrow & 49 \end{array}$$

$$\begin{array}{l|l} \textcircled{1} & 2a = 4 \Rightarrow a = 2 \\ \textcircled{2} & 3a + b = 6 \Rightarrow 3(2) + b = 6 \Rightarrow b = 0 \\ \textcircled{3} & a + b + c = 1 \Rightarrow 2 + 0 + c = 1 \Rightarrow c = -1 \\ & an^2 + bn + c \\ & 2n^2 + 0n - 1 \\ & \therefore 2n^2 - 1 \end{array}$$

$$\begin{array}{c} \text{.....} \\ 2n^2 - 1 \\ \text{.....} \end{array} \quad (3)$$

(Total 5 marks)



# Dependent Probability Trees

15. Fiza has 10 coins in a bag.  
There are three £1 coins and seven 50 pence coins.

Fiza takes at random, 3 coins from the bag.

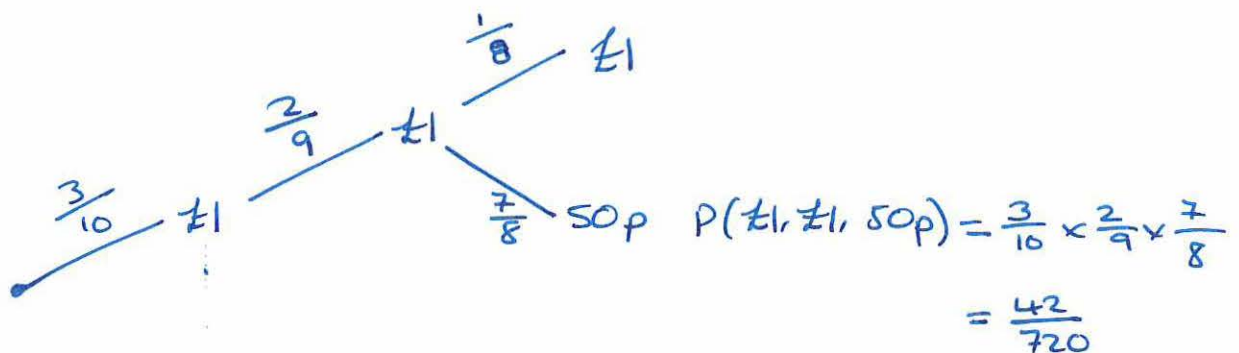
Work out the probability that she takes exactly £2.50.

How many ways to make £2.50?

$$(\text{£1}, \text{£1}, 50\text{p}) \text{ or } (\text{£1}, 50\text{p}, \text{£1}) \text{ or } (50\text{p}, \text{£1}, \text{£1})$$

$$= 3 \times P(\text{£1}, \text{£1}, 50\text{p})$$

$$P(\text{£1}, \text{£1}, 50\text{p}) = \dots$$



Recall that we need all 3 permutations of £1, £1, 50p

$$\therefore 3 \times \frac{42}{720} = \underline{\underline{\frac{126}{720}}}$$

$$\frac{126}{720}$$

(Total 4 marks)

## Direct Proportion

16.  $M$  is directly proportional to  $L^3$ .

When  $L = 2$ ,  $M = 160$

Find the value of  $M$  when  $L = 3$

Direct Proportion	$M \propto L^3$		$K = 20$	$M = 20L^3$
	$M = KL^3$	$\longrightarrow$		
$L = 2, M = 160$	$160 = K(2)^3$		$L = 3$	$M = 20(3)^3$
	$160 = 8K$			$M = 20 \times 27$
$(\div 8)$	$20 = K$	$\nearrow$		$M = \underline{\underline{540}}$

540

(Total 4 marks)

17. Solve  $(x-1)^2 - 2(x-1) - 3 = 0$

Expand	$(x-1)(x-1) - 2(x-1) - 3 = 0$
...	$x^2 - x - x + 1 - 2(x-1) - 3 = 0$
...	$x^2 - x - x + 1 - 2x + 2 - 3 = 0$
collect	$x^2 - 4x = 0$
factorise	$x(x-4) = 0$
solve	$x = 0$ <u>or</u> $x = 4$

.....  
(Total 4 marks)

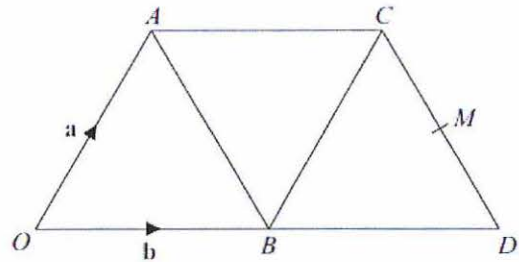
# Vectors

18.  $OACD$  is a trapezium made from three equilateral triangles.

$$\overrightarrow{OA} = \mathbf{a}$$

$$\overrightarrow{OB} = \mathbf{b}$$

$M$  is the midpoint of  $CD$ .



- (a) Write  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= -\mathbf{a} + \mathbf{b}\end{aligned}$$

$$-\mathbf{a} + \mathbf{b}$$

(1)

- (b) Show that  $\overrightarrow{OC}$  is parallel to  $\overrightarrow{BM}$ .

Parallel if scalar multiples of each other

$$\overrightarrow{CO} = \overrightarrow{AB}$$

Expand

collect

Factorise

$$\overrightarrow{OC} = \lambda \overrightarrow{BM}$$

Conclusion

$$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OB} + \overrightarrow{BC} \\ &= \mathbf{b} + \mathbf{a}\end{aligned}$$

$$\begin{aligned}\overrightarrow{BM} &= \overrightarrow{BC} + \overrightarrow{CM} \\ &= \mathbf{a} + \frac{1}{2} \overrightarrow{CD} \\ &= \mathbf{a} + \frac{1}{2} \overrightarrow{AB} \\ &= \mathbf{a} + \frac{1}{2} (-\mathbf{a} + \mathbf{b}) \\ &= \mathbf{a} - \frac{1}{2} \mathbf{a} + \frac{1}{2} \mathbf{b} \\ &= \frac{1}{2} \mathbf{a} + \frac{1}{2} \mathbf{b} \\ &= \frac{1}{2} (\mathbf{a} + \mathbf{b})\end{aligned}$$

$$\overrightarrow{OC} = 2\overrightarrow{BM}$$

$\therefore \overrightarrow{OC}$  is parallel to  $\overrightarrow{BM}$

(4)

(Total 5 marks)

19. Prove algebraically that the sum of the squares of two consecutive integers is always an odd number.

Integer = $n$	$(n+1)^2 + (n)^2$
Consecutive = $n+1$	
Expand	$= (n+1)(n+1) + n^2$
Expand	$= n^2 + n + n + 1 + n^2$
Collect	$= 2n^2 + 2n + 1$
Factorise	$= 2(n^2 + n) + 1$
Conclusion	$2n$ is even $\therefore 2(n^2 + n)$ is even $\therefore 2(n^2 + n) + 1$ is odd. <span style="float: right;">□</span>

(Total 3 marks)

# Rationalising Surds

20. Given that  $\frac{8-\sqrt{18}}{\sqrt{2}} = a + b\sqrt{2}$ , where  $a$  and  $b$  are integers,

find the value of  $a$  and the value of  $b$ .

$$\sqrt{18} = \sqrt{9 \times 2} \quad \therefore \frac{8-3\sqrt{2}}{\sqrt{2}}$$

Rationalise  
 $\times \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$$\sqrt{2} \times \sqrt{2} = 2$$

$$\sqrt{2} \times 3\sqrt{2} = 3 \times 2 \text{ expand}$$

$$= \frac{8-3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}(8-3\sqrt{2})}{2}$$

$$= \frac{8\sqrt{2} - 6}{2}$$

$$= \underline{\underline{4\sqrt{2} - 3}}$$

$$a = \underline{\underline{-3}}$$

$$b = \underline{\underline{4}}$$

(Total 3 marks)

TOTAL FOR PAPER IS 80 MARKS