

# GCSE Mathematics

## Practice Tests: Set 3

### Paper 2H (Calculator)

Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator.

#### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**



#### Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

#### Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1.

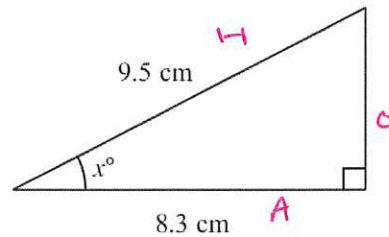


Diagram NOT accurately drawn

Work out the value of  $x$ .  
Give your answer correct to 1 decimal place.

SIGNIFICANT

$$\cos \theta = \frac{A}{H}$$

$$\cos x = \frac{8.3}{9.5}$$

$$x = 29.1 \text{ (1 d.p.)}$$

Shift cos

$$x = 29.1$$

(Total 3 marks)

2. On July 1st 2004, Jack invested £2000 at 5% per annum compound interest.

Repeated % change

Work out the value of Jack's investment on July 1st 2006

$$\text{Start} \times \text{multiplier}^t = \text{End}$$

$$\text{multiplier} = 1 + i\%$$

$$= 1.05$$

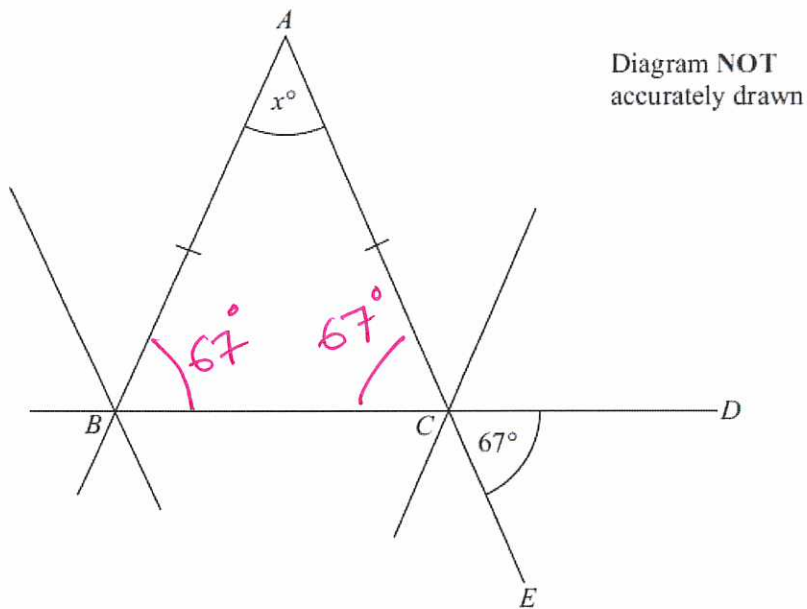
$$£2000 \times 1.05^2 = £2205$$

$$£ 2205$$

(Total 3 marks)

# Angles in Triangles

3. The diagram shows part of the design of a stained glass window.



$ABC$  is an isosceles triangle.  $BCD$  and  $ACE$  are straight lines. Angle  $DCE = 67^\circ$ .

Work out the size of the angle marked  $x^\circ$ . Give reasons for your answer.

$$\begin{aligned}\hat{ACB} &= \hat{DCE} = 67^\circ \\ \hat{ACB} &= \hat{ABC} = 67^\circ \\ \hat{BAC} &= 180^\circ - 67^\circ - 67^\circ \\ &= 46^\circ\end{aligned}$$

Vertically opposite angles are equal.  
Base angles in isosceles triangle equal  
Angles in a triangle =  $180^\circ$

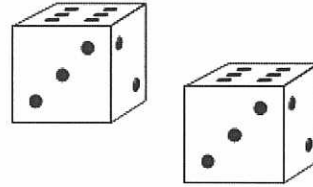
(Total 4 marks)

# Independent Event Probability

4. Naomi is playing a board game.  
She must throw two fair dice.

She must get a 6 on each dice to start the game.

Work out the probability that she will not start the game on her first throw.



$$P(\text{Roll a 6}) = \frac{1}{6}$$

$$P(\text{Roll a 6, Roll a 6}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$\begin{aligned} P(\text{NOT 6 and 6}) &= 1 - \frac{1}{36} \\ &= \frac{35}{36} \end{aligned}$$

.....  
**(Total 3 mark)**

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# Direct Proportion

5. A company sells circular mirrors.  
The price  $P$  pounds of a mirror is proportional to the square of its radius  $r$  cm.

A mirror with radius 20 cm has a price of £36

Find a formula for  $P$  in terms of  $r$ .

<p>Direct Proportion</p> <p><math>P = 36, r = 20</math></p> <p><math>(\div 400)</math></p>	<p><math>P \propto r^2</math></p> <p><math>P = kr^2</math></p> <p><math>36 = k(20)^2</math></p> <p><math>36 = 400k</math></p> <p><math>\frac{36}{400} = k</math></p>	<p><math>\longrightarrow</math></p> <p><math>P = \frac{36r^2}{400} = \frac{9r^2}{100}</math></p>
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$P = \frac{9r^2}{100}$

(Total 3 marks)

6. The width of a rectangle is a whole number of centimetres.  
The length of the rectangle is 9 cm longer than its width.

The perimeter of the rectangle is less than 200 cm.

Find the greatest possible width of the rectangle.



<p><math>P &lt; 200</math></p> <p>collect</p> <p><math>(-18)</math></p> <p><math>(\div 4)</math></p>	<p>Perimeter <math>&lt; 200</math></p> <p><math>x + x + 9 + x + x + 9 &lt; 200</math></p> <p><math>4x + 18 &lt; 200</math></p> <p><math>4x &lt; 182</math></p> <p><math>x &lt; 45.5</math></p>	<p>..... cm</p>
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$x$  is a whole number      width (max) = 45

(Total 4 marks)

# 3D Pythagoras

7. The diagram shows Diana's suitcase.  
The suitcase is in the shape of a cuboid.

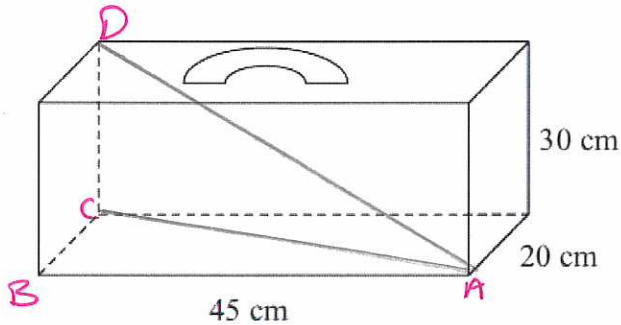


Diagram NOT accurately drawn

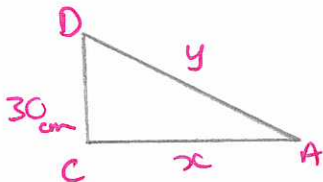
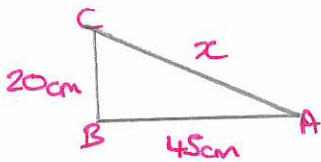
NEED diagonal from top to bottom.

Diana has a walking stick that folds.  
The folded walking stick has a length of 60 cm.

Diana wants to put the folded walking stick in the suitcase.

Will the folded walking stick fit in the suitcase?

Two triangles



$$\begin{aligned} \text{Pythagoras: } a^2 + b^2 &= c^2 \\ 20^2 + 45^2 &= c^2 \\ 2425 &= c^2 \\ \sqrt{2425} &= c = 49.24... = x \end{aligned}$$

$$\begin{aligned} \text{Pythagoras: } a^2 + b^2 &= c^2 \\ 30^2 + x^2 &= y^2 \\ \text{from ANS. } \rightarrow 30^2 + 2425 &= y^2 \\ 3325 &= y^2 \\ \sqrt{3325} &= y = 57.66 \text{ cm} \dots \end{aligned}$$

conclusion:

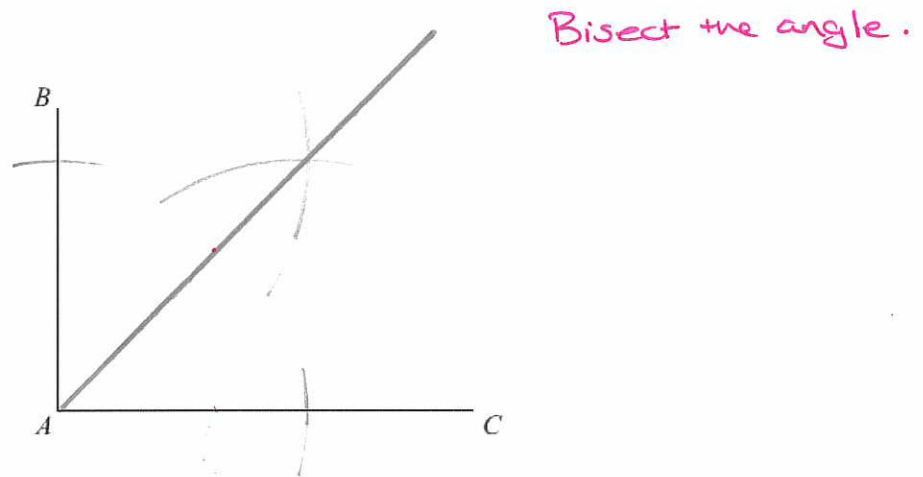
Stick is 60cm. This is too big to fit since  $60 > 57.66$ .

∴ No

(Total 4 marks)



8. Draw the locus of all points which are equidistant from the lines  $AB$  and  $AC$ .



(Total 2 marks)

9. In a sale normal prices are reduced by 20%.

Reverse Percentages

A washing machine has a sale price of £464

By how much money is the normal price of the washing machine reduced?

$$\begin{array}{l|l} & \text{£464} = 100\% - 20\% = 80\% \\ (\div 80) & \text{£5.80} = 1\% \\ (\times 100) & \text{£580} = 100\% \end{array}$$

Reduction  $\text{£580} - \text{£464} = \underline{\underline{\text{£116}}}$

£ 116 .....

(Total 3 marks)

10. The surface area of Earth is  $510\,072\,000\text{ km}^2$ .  
The surface area of Jupiter is  $6.21795 \times 10^{10}\text{ km}^2$ .

The surface area of Jupiter is greater than the surface area of Earth.

How many times greater?

Give your answer in standard form.

"How many earths in a Jupiter" → DIVISION

$$\therefore \frac{6.21795 \times 10^{10}}{510072000}$$

$$= 121.9033783$$

$$\approx 122$$

$$= \underline{\underline{1.22 \times 10^2}}$$

$$\dots\dots\dots 1.22 \times 10^2$$

(Total 3 marks)

USE YOUR CALCULATOR

11. 25 students in class A did a science exam.  
30 students in class B did the same science exam.

The mean mark for the 25 students in class A is 67.8.

The mean mark for all the 55 students is 72.0.

Work out the mean mark for the students in class B.

Missing Mean

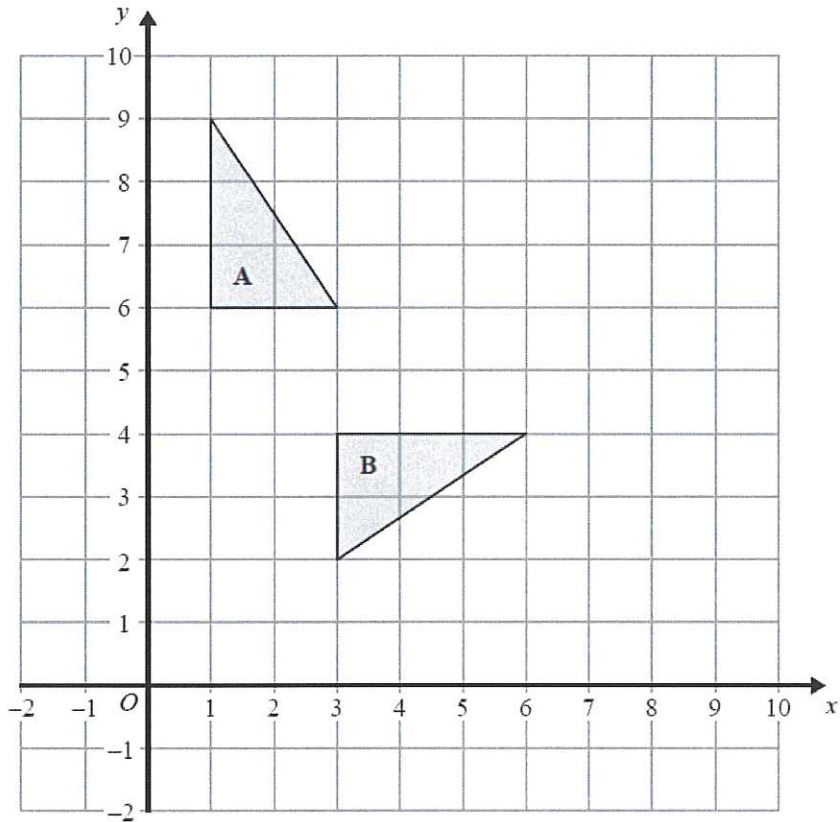
Total marks A	$25 \times 67.8 = 1695$
Total marks <del>A</del> B	$55 \times 72 = 3960$
Total marks B	$3960 - 1695 = 2265$
mean marks B	$2265 \div 30 = \underline{\underline{75.5}}$

$$\dots\dots\dots 75.5$$

(Total 3 marks)



12.



Describe fully the single transformation that maps triangle A onto triangle B.

Rotation by  $90^\circ$  clockwise about centre (1,4)

.....  
 .....

(Total 3 marks)



mass density volume

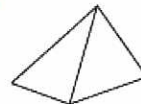
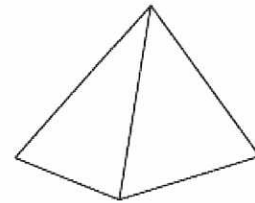
13. Fred has a solid brass model of an Egyptian pyramid.

The model has a volume of  $3000 \text{ cm}^3$ .

The density of the brass is  $8.5 \text{ g/cm}^3$ .

(a) Calculate the mass of the model.  
Give your answer in kg.

Diagram NOT accurately drawn



Model

Egyptian pyramid

$$m = D \times V \quad | \quad m = 3000 \text{ cm}^3 \times 8.5 \text{ g/cm}^3$$

$$= 25500 \text{ g}$$

$$= 25.5 \text{ kg}$$

convert g to kg  
( $\div 1000$ )

..... 25.5 kg  
(2)

The model and the Egyptian pyramid are mathematically similar. The length of the base of the model is 25 cm.

The length of the base of the Egyptian pyramid is 225 m.

LSF ASF VSF

(b) Calculate the volume of the Egyptian pyramid.  
Give your answer in  $\text{m}^3$ .

Length scale factor  
( $\div 25$ )

Model : Real  
25 cm : 225 m  
1 cm : 9 m

$VSF = (LSF)^3$

1 cm : 729 m  $\therefore$  volume is 729 bigger.

Volume of model  
from (a) =  $3000 \text{ cm}^3$

$\downarrow \times 3000$        $\downarrow \times 3000$   
 $3000 \text{ cm}^3 : 2187000 \text{ m}^3$

$\therefore 2187000$  .....  $\text{m}^3$

(3)

(Total 5 marks)

14. Here is a rectangular sheet of metal.  
A square hole is cut out of the metal.

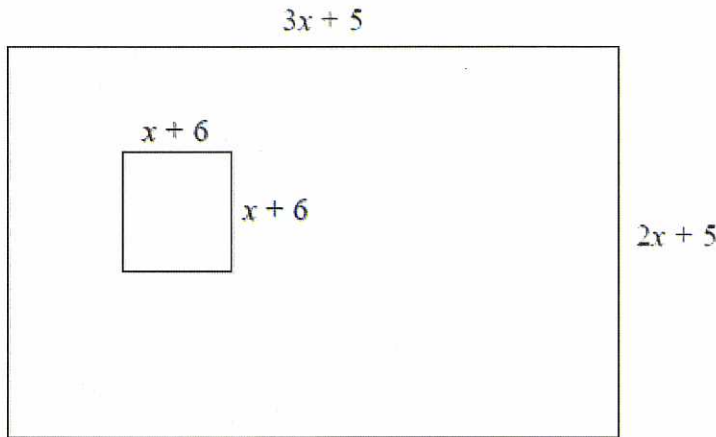


Diagram **NOT**  
accurately drawn

The length of the rectangle is  $3x + 5$   
The width of the rectangle is  $2x + 5$   
The square has sides of length  $x + 6$   
All measurements are in centimetres.

The perimeter of the square hole is  $\frac{3}{5}$  of the perimeter of the rectangle.

Work out the length of a side of the square hole.

$P_{\text{square}} = \frac{3}{5} P_{\text{rectangle}}$ <p style="color: red; margin-left: 20px;">collect</p> <p style="margin-left: 20px;"><math>(-4x)</math></p> <p style="margin-left: 20px;"><math>(-12)</math></p> <p style="margin-left: 20px;"><math>(\div 2)</math></p> <p style="margin-left: 20px;">Square hole length: <math>x + 6</math></p>	$x + 6 + x + 6 + x + 6 + x + 6 = \frac{3}{5} [3x + 5 + 2x + 5 + 3x + 5 + 2x + 5]$ $4x + 24 = \frac{3}{5} [10x + 20]$ $4x + 24 = 6x + 12$ $24 = 2x + 12$ $12 = 2x$ $6 = x$ $\therefore = 6 + 6$	$= \frac{3}{5} [10x + 20]$ $= 6x + 12$ $= 2x + 12$ $= 2x$ $= x$ $= \underline{\underline{12 \text{ cm}}}$ <p style="text-align: right; margin-right: 20px;">..... 12 cm</p> <p style="text-align: right; margin-right: 20px;"><b>(Total 5 marks)</b></p>
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## Expanding Triple Brackets

15. (a) Expand and simplify  $(2x + 1)(x - 3)(x + 5)$

$$\begin{array}{l|l} (x-3)(x+5) & = x^2 + 5x - 3x - 15 \\ \text{collect} & = x^2 + 2x - 15 \\ (x^2 + 2x - 15)(2x + 1) & = 2x^3 + x^2 + 4x^2 + 2x - 30x - 15 \\ \text{collect} & = \underline{2x^3 + 5x^2 - 28x - 15} \end{array}$$

.....  
(3)

(b) Make  $r$  the subject of  $5r + 1 = a(m + r)$

Rearranging Complex Formula

$$\begin{array}{l|l} \text{expand} & 5r + 1 = am + ar \\ (-ar) & 5r - ar + 1 = am \\ (-1) & 5r - ar = am - 1 \\ \text{factorise} & r(5 - a) = am - 1 \\ (\div (5 - a)) & r = \frac{am - 1}{5 - a} \end{array}$$

$$\underline{r = \frac{am - 1}{5 - a}}$$

(3)

(Total 6 marks)

16.

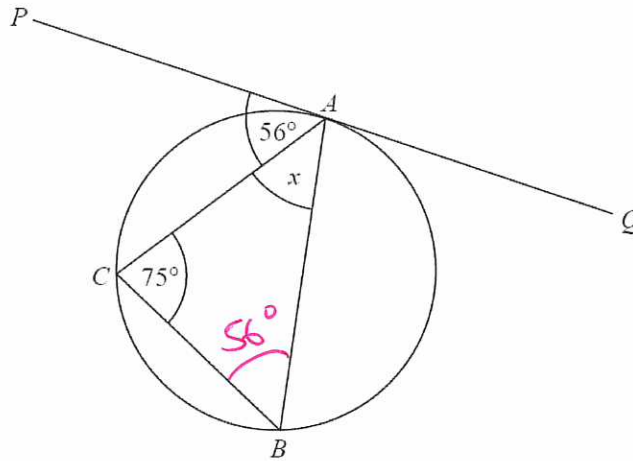


Diagram **NOT** accurately drawn

$A$ ,  $B$  and  $C$  are points on the circumference of a circle.  
 The straight line  $PAQ$  is a tangent to the circle.  
 Angle  $PAC = 56^\circ$   
 Angle  $ACB = 75^\circ$

Work out the size of the angle marked  $x$ .  
 Give reasons for each stage of your working.

$$\begin{array}{l|l} \hat{A}BC = \hat{P}AC = 56^\circ & \text{Alternate segment theorem} \\ \hat{B}AC = x^\circ = 180^\circ - 56^\circ - 75^\circ & 180^\circ \text{ in a triangle} \\ = \underline{49^\circ} & \end{array}$$

(Total 3 marks)

# Advanced Trig with Bearings

17.

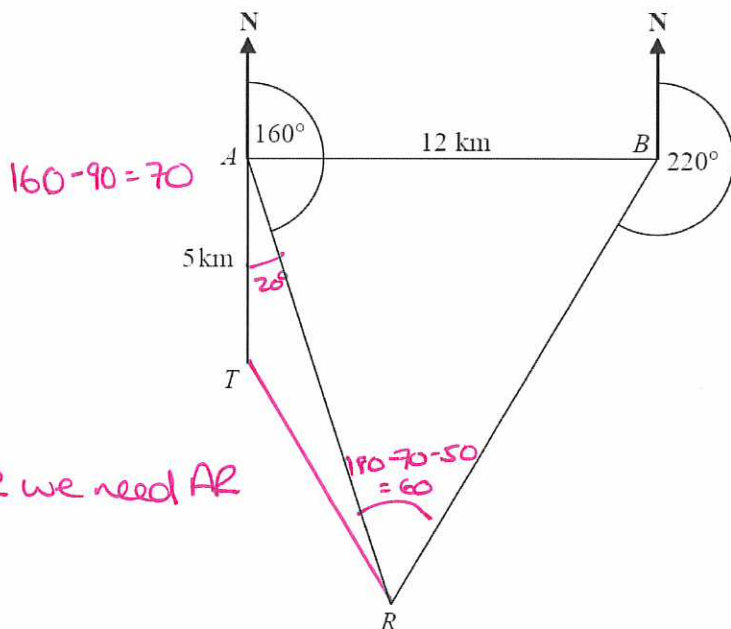
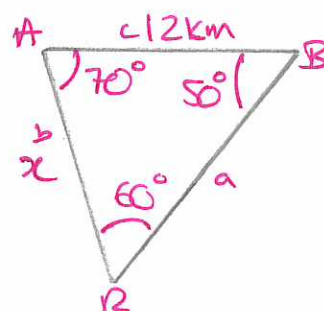


Diagram NOT accurately drawn

$$160 - 90 = 70$$

$$270 - 220 = 50$$

To find TR we need AR



There is a coastguard station at point  $A$  and at point  $B$ .  
 $B$  is due East of  $A$ .  
 The distance from  $A$  to  $B$  is 12 km.

There is a rowing boat at point  $R$ .  
 $R$  is on a bearing of  $160^\circ$  from  $A$ .  
 $R$  is on a bearing of  $220^\circ$  from  $B$ .

There is a speedboat at point  $T$ .  
 $T$  is 5 km due South of  $A$ .

Work out the shortest distance from  $T$  to  $R$ .  
 Give your answer correct to 1 decimal place.  
 You must show all your working.

Sine Rule

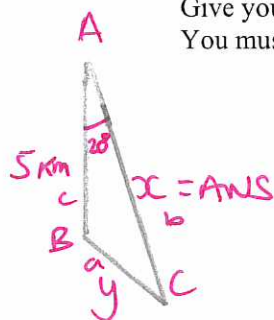
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

( $\times \sin(50)$ )

$$\therefore \frac{x}{\sin 50} = \frac{12}{\sin 60}$$

$$x = \frac{12}{\sin 60} \times \sin 50$$

$$x = 10.6146... \\ = \text{ANS}$$



cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$\sqrt{\text{ANS}}$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (\text{ANS})^2 + (5)^2 - 2(\text{ANS})(5) \cos(20)$$

$$a^2 = 37.925...$$

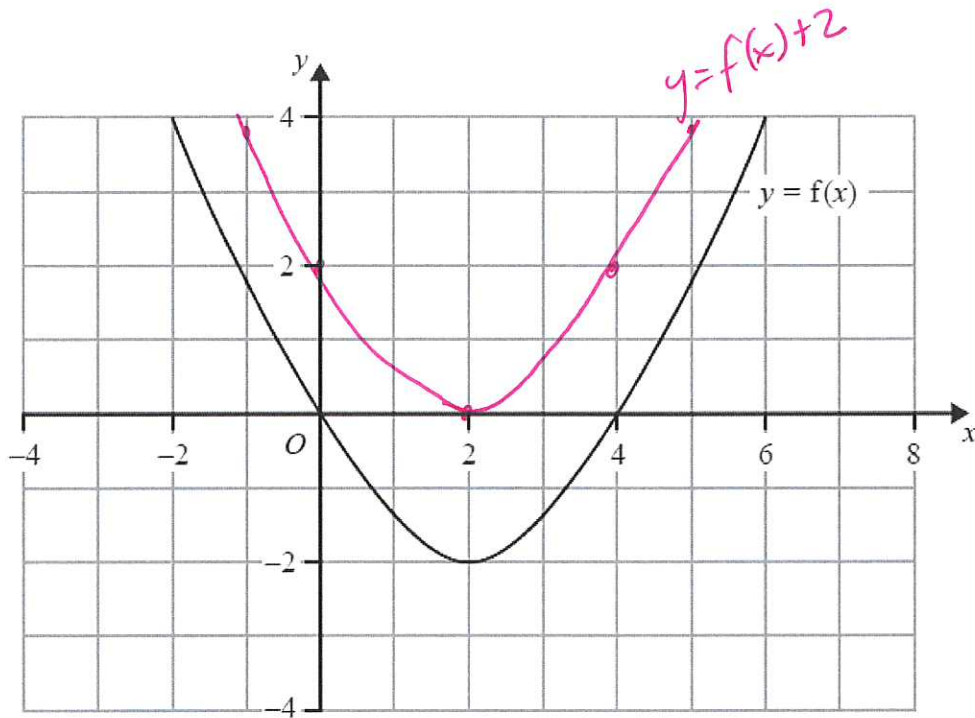
$$a = 6.158359716$$

$$a = 6.2 \text{ (1 d.p.)} \\ \text{km}$$



# Transformations of Functions

18. The diagram shows part of the curve with equation  $y = f(x)$ .



- (a) (i) Write down the coordinates of the points where the graph of  $y = f(x - 2)$  crosses the  $x$ -axis.

Translated by  $\begin{pmatrix} 2 \\ 0 \end{pmatrix} \rightarrow$

( 6, 0 ) and ( 2, 0 )

- (ii) Write down the coordinates of the point where the graph of  $y = f(x - 2)$  crosses the  $y$ -axis.

( 0, 4 )

(2)

- (b) On the diagram above, sketch the graph of  $y = f(x) + 2$

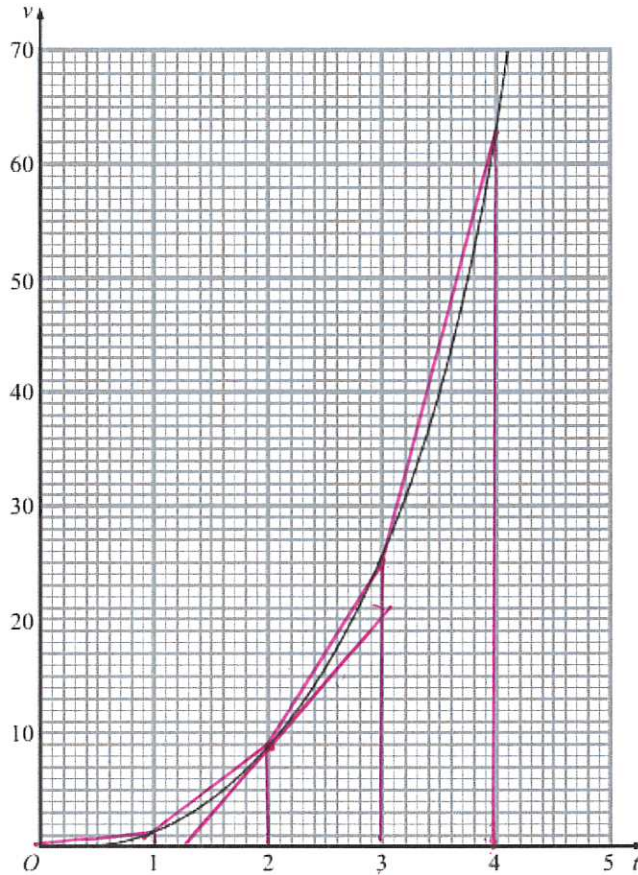
Translated by  $\begin{pmatrix} 0 \\ 2 \end{pmatrix} \uparrow$

(2)

(Total 4 marks)

# Velocity Time Graphs

19. The graph shows the velocity,  $v$  metres per second, of a rocket at time  $t$  seconds.



(a) Find an estimate for the rate of change of the velocity of the rocket at  $t = 2$

Draw tangent

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{20 - 8}{3 - 2} = \frac{12}{1} = \underline{\underline{12}}$$

..... 12 m/s<sup>2</sup>

(3)

(b) Find an estimate for the distance travelled by the rocket in the first 4 seconds.

Use 4 strips of equal width Area under the curve.

① $\frac{b \times h}{2}$	① $A = \frac{1 \times 1}{2} = 0.5 \text{ m}$	
② $\frac{h(a+b)}{2}$	② $A = \frac{1(1+8)}{2} = 4.5 \text{ m}$	..... 66.5 m
③ $\frac{h(a+b)}{2}$	③ $A = \frac{1(8+20)}{2} = 14 \text{ m}$	(3)
④ $\frac{h(a+b)}{2}$	④ $A = \frac{1(26+62)}{2} = 44 \text{ m}$	(Total 6 marks)

Total Distance Total:  $0.5 \text{ m} + 4.5 \text{ m} + 14 \text{ m} + 44.5 \text{ m} = \underline{\underline{66.5 \text{ m}}}$

20. Rhys has a beehive.

The number of bees in the beehive is decreasing.

Rhys counts the number of bees in the hive at the start of week 5. He counts the number of bees in the hive at the start of week 7.

Here are his results.

week	number of bees
5	1200
7	900

Assuming that the population of bees is decreasing exponentially, how many bees were there at the start of week 2?

You must show your working.

Start  $\times$  multiplier<sup>t</sup> = End  
 ( $\div 1200$ )  
ANS

$$1200 \times \text{multiplier}^2 = 900$$

$$\text{multiplier}^2 = \frac{900}{1200} = 0.75$$

$$\text{multiplier} = \frac{\sqrt{3}}{2}$$

Now we need to work backwards, starting at week 5, to week 2.

Normally  $\times$  multiplier, but we now  $\div$  multiplier.  
 Since we are going back in time to week 2 (3 weeks)

$$1200 \div \left(\frac{\sqrt{3}}{2}\right)^3 = 1847.520861$$

$$\approx 1848 \text{ bees}$$

1848 bees

(Total 5 marks)

# Area of a Trapezium

21. A trapezium  $ABCD$  has an area of  $5\sqrt{6} \text{ cm}^2$ .

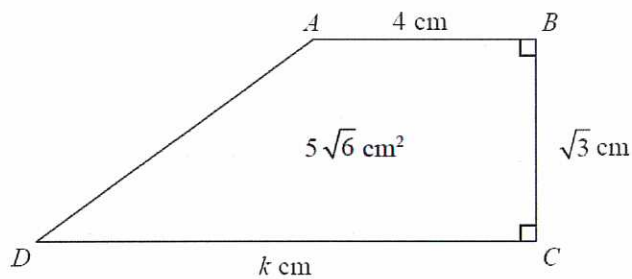


Diagram NOT accurately drawn

$$\begin{aligned} AB &= 4 \text{ cm.} \\ BC &= \sqrt{3} \text{ cm.} \\ DC &= k \text{ cm.} \end{aligned}$$

Calculate the value of  $k$ , giving your answer in the form  $a\sqrt{b} - c$ , where  $a$ ,  $b$  and  $c$  are positive integers. Show each step in your working.

$$\begin{array}{l|l} A = \frac{h(a+b)}{2} & 5\sqrt{6} = \frac{\sqrt{3}(4+k)}{2} \\ ( \times 2 ) & 10\sqrt{6} = \sqrt{3}(4+k) \\ ( \div \sqrt{3} ) & 10\sqrt{2} = 4+k \\ ( -4 ) & 10\sqrt{2} - 4 = k \end{array}$$

$$k = 10\sqrt{2} - 4$$

(Total 3 marks)

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**TOTAL FOR PAPER IS 80 MARKS**