ms ee4

# GCSE Mathematics Practice Tests: Set 2

## Paper 3H (Calculator)

### Time: 1 hour 30 minutes

You should have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator.

**Instructions**

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  *– there may be more space than you need.*
- **Calculators may be be used.**
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

**Information**

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  *– use this as a guide as to how much time to spend on each question.*

**Advice**

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Reverse Percentages

**Answer ALL questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1.** Each year Wenford Hospital records how long patients wait to be treated in the Accident and Emergency department.

In 2015 patients waited 11% less time than in 2014.
In 2015 the average time patients waited was 68 minutes.

(a) Work out the average time patients waited in 2014.
Give your answer to the nearest minute.

2015:

100% − 11% = 89%

89% = 68 minutes

(÷ 89) 1% = 0.764...

(×100) 100% = 76.4

Rounding = 76 minutes (nearest minute) minutes

**(3)**

The hospital has a target to reduce the average time patients wait to be treated in the Accident and Emergency department to 60 minutes in 2016.

(b) Work out the percentage decrease from 68 minutes to 60 minutes.

Percentage Change

% change

$\% \text{ change} = \frac{\text{change}}{\text{original}} (\times 100)$

$= \frac{8}{68} (\times 100)$

$= 11.7647\ldots \simeq 11.8\%$

11.8 %

**(2)**

**(Total 5 marks)**

**2.** There are only red pens and blue pens in a box.
There are 12 red pens in the box.

The probability of taking at random a blue pen from the box is $\frac{2}{3}$

Work out the total number of pens in the box.

| | |
|---|---|
| Proportion: | Blue $= \frac{2}{3}$ Red $= \frac{1}{3}$ |
| Quantity | $= 24$ $= 12$ |
| Total | Blue + Red $= 24 + 12$ <br> $= \underline{\underline{36}}$ |

36 ..................................................

**(Total 3 marks)**

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**3.** Each length of the side of square B is twice the length of the side of square A. LSF ASF VSF

John says that this means the area of square B is twice the area of square A.

Is John right?
Justify your answer.

No since area scale factor = (length scale factor)$^2$

..................................................

**(Total 1 mark)**

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Mixed Number operations

**4.** Show that $7\frac{1}{2} - 4\frac{2}{3} = 2\frac{5}{6}$

Improper fractions | $\frac{15}{2} - \frac{14}{3}$

LCM = 6 | $= \frac{45}{6} - \frac{28}{6}$

$= \frac{17}{6}$

Mixed Number | $= \underline{\underline{2\frac{5}{6}}}$ $\square$

**(Total 3 marks)**

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**5.** Make $t$ the subject of $5(t - g) = 2t + 7$

Rearranging Formulae

expand | $5t - 5g = 2t + 7$

$(+5g)$ | $5t = 2t + 7 + 5g$

$(-2t)$ | $3t = 7 + 5g$

$(\div 3)$ | $t = \frac{7 + 5g}{3}$

$t = \frac{7+5g}{3}$ ..............................

**(Total 3 marks)**

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money problem

**6.** Henry is thinking about having a water meter.

These are the two ways he can pay for the water he uses.

**Water Meter**

A charge of £28.20 per year

**plus**

91.22p for every cubic metre of water used

**1 cubic metre = 1000 litres**

**No Water Meter**

A charge of £107 per year

Henry uses an average of 180 litres of water each day.

Henry wants to pay as little as possible for the water he uses.
Should Henry have a water meter?

| | |
|---|---|
| water per year | $365 \times 180\text{l} = 65700$ litres |
| cubic metres per year | $65700 \div 1000 = 65.7\text{cm}^3$ |
| cost of water (pence) | $65.7 \times 91.22\text{p} = 5993.154\text{p}$ |
| cost of water (£) | $= £59.93$ |
| Total cost | $£28.20 + £59.93 = £88.13$ |
| Conclusion | Get a water meter since £88.13 < £107. |

**(Total 5 marks)**

**7.** Cameron invests £1200 for 3 years in a savings account.
He gets 4.1% per annum **simple** interest.

Mitchell invests £1200 for 3 years in a savings account.
He gets 4% per annum **compound** interest.

Who will have the most money in his savings account at the end of the 3 years?
You must show all your working.

Cameron

100% + 4.1% = 104.1%
= 1.041

£1200 × 1.041 = £1249.20

∴ Interest gives £49.20 per year

£49.20 × 3 = £147.60

£1200 + £147.60 = £1347.60

Mitchell

100% + 4% = 104%
= 1.04

$£1200 \times 1.04^3 = £1349.84$

∴ Mitchell has the most money in their account.

**(Total 5 marks)**

Linear Sequences

**8.** Here are the first four terms of an arithmetic sequence.

3 10 17 24

(a) Find, in terms of *n*, an expression for the *n*th term of this arithmetic sequence.

$$=-4 \xleftarrow{-7} 3 \xrightarrow{+7} 10 \xrightarrow{+7} 17 \xrightarrow{+7} 24$$

$7n-4$

**(2)**

(b) Is 150 a term of this sequence?

You must explain how you get your answer.

$7n-4=150$

$(+4)$ $7n = 154$

$(\div 7)$ $n = 22$

Yes, 150 is in the sequence – the 22nd term.

**(2)**

**(Total 4 marks)**

Averages from Raw Data

**9.** Here are the marks that James scored in eleven maths tests.

16 12 19 18 17 13 13 20 11 19 17

(*a*) Find the interquartile range of these marks.

IQR = UQ − LQ

Re-order: 11 12 13 13 16 17 17 18 19 19 20

$LQ = \frac{n+1}{4} = 3^{rd}$ term $= 13$

$UQ = \frac{3(n+1)}{4} = 9^{th}$ term $= 19$

$IQR = 19 - 13$

$= 6$

6

**(3)**

Sunil did the same eleven maths tests.
The median mark Sunil scored in his tests is 17.
The interquartile range is 8.

(*b*) Which one of Sunil or James has the more consistent marks?
Give a reason for your answer.

James, since the spread of data is less. (smaller IQR).

**(1)**

Sunil did four more maths tests.
His scores in these four tests were 16, 20, 18 and 10.

(*c*) How does his new median mark for the fifteen tests compare with his median mark of 17 for the eleven tests?

Tick (✓) one box.

☐ new median is lower ☑ new median is 17 ☐ new median is higher

Explain your answer.

median is the same since there are two pieces of data below and two above 17.

**(1)**

**(Total 5 marks)**

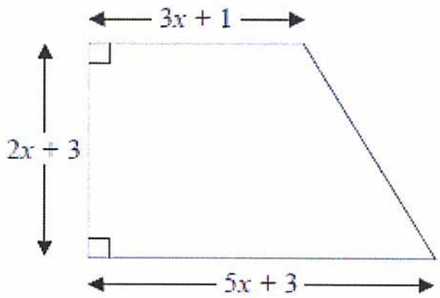
Forming and Solving Quadratics

$a = 3x+1$

$b = 5x+3$

$h = 2x+3$

**10.** The diagram shows a trapezium.




All the measurements are in centimetres.
The area of the trapezium is 46 cm$^2$.

(a) Show that $x^2 + 2x - 5 = 0$

$A = \frac{h(a+b)}{2}$

$A = \frac{(2x+3)(3x+1+5x+3)}{2}$

$46 = \frac{(2x+3)(3x+1+5x+3)}{2}$

$(\times 2)$ $92 = (2x+3)(3x+1+5x+3)$

collect $92 = (2x+3)(8x+4)$

expand $92 = 16x^2+8x+24x+12$

collect $92 = 16x^2+32x+12$

$(-92)$ $0 = 16x^2+32x-80$

$(\div 16)$ $0 = x^2+2x-5$ $\square$

**(3)**

(b) Solve the equation $x^2 + 2x - 5 = 0$
Give your solutions correct to 2 decimal places.

Quadratic Formulae

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=1$
$b=2$
$c=-5$

$x = \frac{-(2) \pm \sqrt{(2)^2-4(1)(-5)}}{2(1)}$

$x_+ = 1.45$ (2d.p)

$x_- = -3.45$ (2d.p)

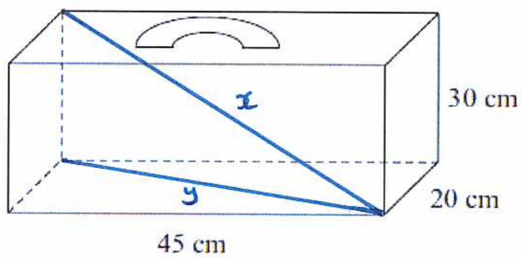
$x = 1.45$ or $x = -3.45$ ..........................................

**(3)**

**(Total 6 marks)**

3D Pythagoras

**11.** The diagram shows Diana's suitcase.
The suitcase is in the shape of a cuboid.

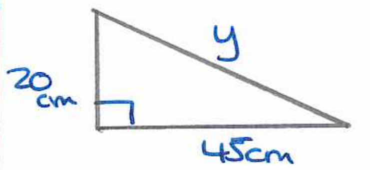


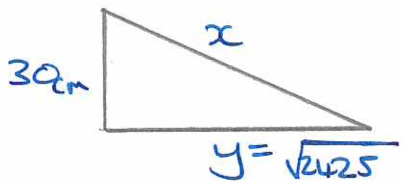

Diana has a walking stick that folds.
The folded walking stick has a length of 60 cm.

Diana wants to put the folded walking stick in the suitcase.

Will the folded walking stick fit in the suitcase?

Two Triangles




Pythagoras
$a^2+b^2=c^2$

$20^2+45^2=c^2=y^2$

$2425 = y^2$

$\sqrt{ANS}$

$\sqrt{2425} = y$

$30^2+(\sqrt{2425})^2=x^2$

$3325 = x^2$

$\sqrt{3325} = x = 57.66$ (2dp)

Conclusion

No, the stick will not fit since the longest possible dimension is 57.66 and 57.66 cm < 60cm

**(Total 4 marks)**

**12.** The surface area of Earth is 510 072 000 km$^2$.
The surface area of Jupiter is $6.21795 \times 10^{10}$ km$^2$.

The surface area of Jupiter is greater than the surface area of Earth.
How many times greater?
Give your answer in standard form.

"How many x Jupiter bigger than Earth?"

Division

$$\frac{6.21795 \times 10^{10}}{510072000}$$

*USE YOUR CALCULATOR*

$= 121.9033783$

Standard Form

$\simeq 1.22 \times 10^2$

$1.22 \times 10^2$ ..........................................

**(Total 3 marks)**

Inverse Proportion

**13.** Brian's band is playing at a concert in a hall.

The loudness of a band varies inversely as the square of the distance from the band.
Brian measures the normal loudness of his band as 100 decibels at a distance of 5 metres.

The band has to stop playing if the loudness is 85 decibels or more at a distance of 5.4 metres.

Does the band have to stop playing?

Inverse Proportion

$L \propto \frac{1}{D^2}$

$L = \frac{k}{D^2}$

Substitute

$(100) = \frac{k}{(5)^2}$

$100 = \frac{k}{25}$

(×2500)

$2500 = k$

$k = 2500$

$D = 5.4$

$L = \frac{2500}{D^2}$

$L = \frac{2500}{(5.4)^2}$

$L = 85.73...$

Conclusion

Yes, the band must stop playing since their loudness is above 85.

**(Total 4 marks)**

Circle Theorems

**14.**

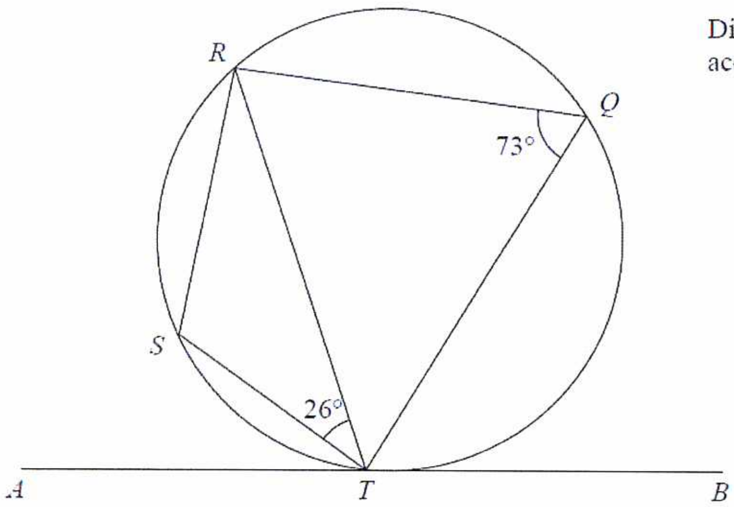



Diagram **NOT** accurately drawn

$Q$, $R$, $S$ and $T$ are points on a circle.
$ATB$ is the tangent to the circle at $T$

Angle $STR = 26°$
Angle $RQT = 73°$

Work out the size of angle $STA$
Give a reason for each stage in your working.

$R\hat{Q}T = A\hat{T}R = 73°$ | Alternate segment theorem

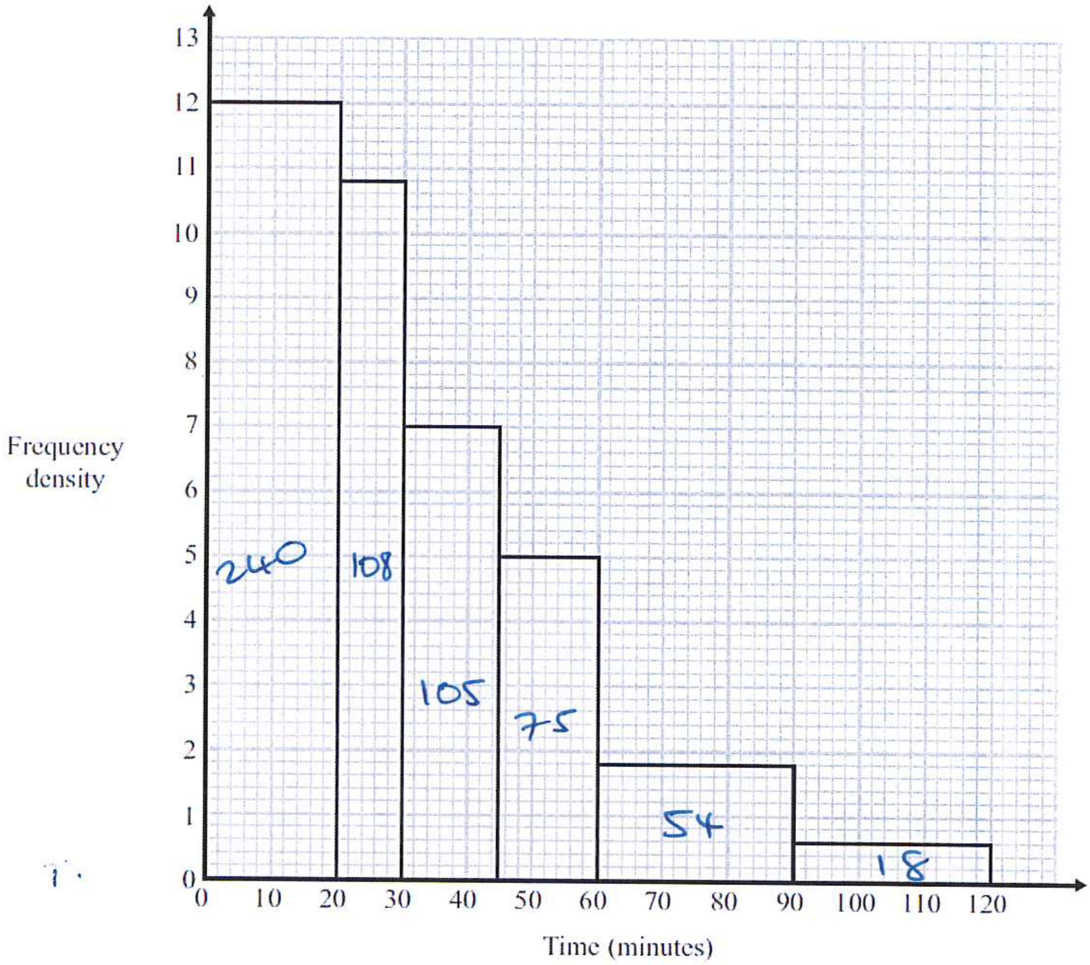
$A\hat{T}S = 73° - 26°$
$= \underline{\underline{47°}}$

47 °

**(Total 3 marks)**

Histograms

**15.** The histogram shows information about the times, in minutes, that some passengers had to wait at an airport.




Work out the percentage of the passengers who had to wait for more than one hour.

$f = c.w \times f.d.$

1 hour or less:

$20 \times 12 = 240$
$10 \times 10.8 = 108$
$15 \times 7 = 105$
$15 \times 5 = 75$
$= 528$

more than 1 hour:

$30 \times 1.8 = 54$
$30 \times 0.6 = 18$
$72$

Percentage 1 hour +

$= \frac{72}{528+72} (\times 100) = 12\%$

.............................12 %

**(Total 3 marks)**

Index Laws
(change of base)

**16.** Given that $\left(2^{\frac{1}{2}}\right)^n = \dfrac{2^x}{8^y}$

express $n$ in terms of $x$ and $y$.

$a^{-m} = \frac{1}{a^m}$

$8 = 2^3$

$(a^m)^n = a^{m \times n}$

$a^{\cancel{m+n}} = a^m \times a^n$

$(a^m)^n = a^{m \times n}$

$(\times 2)$

$\frac{2^x}{8^y} = 2^x \times 8^{-y}$

$= 2^x \times (2^3)^{-y}$

$= 2^x \times 2^{-3y}$

$= 2^{x-3y}$

Also: $\left(2^{\frac{1}{2}}\right)^n = 2^{\frac{n}{2}}$

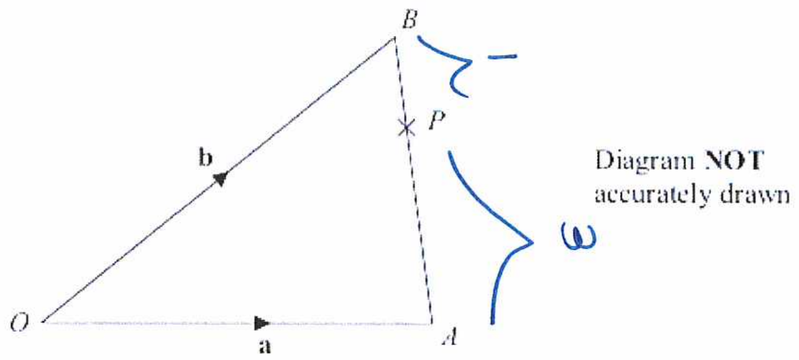
$\therefore \quad \frac{n}{2} = x - 3y$

$n = 2x - 6y$ ..........................................................

**(Total 3 marks)**

Vectors

**17.**




*OAB* is a triangle.

$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -\underline{a} + \underline{b}$

$-\underline{a} + \underline{b}$ ..............................................

**(1)**

*P* is the point on *AB* such that *AP* : *PB* = 3 : 1

(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$
$= \underline{a} + \frac{3}{4}(\overrightarrow{AB})$
using (a) $= \underline{a} + \frac{3}{4}(-\underline{a} + \underline{b})$
expand $= \underline{a} - \frac{3}{4}\underline{a} + \frac{3}{4}\underline{b}$
collect $= \frac{1}{4}\underline{a} + \frac{3}{4}\underline{b}$
factorise $= \frac{1}{4}(\underline{a} + 3\underline{b})$

$\frac{1}{4}(\underline{a} + 3\underline{b})$ ..............................................

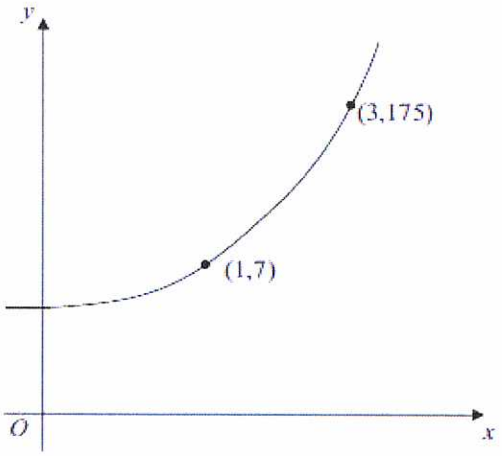
**(3)**

**(Total 4 marks)**

Exponential Graphs

Simultaneous Equations

**18.**




The sketch shows a curve with equation

$y = ka^x$ ← Exponential Graph

where $k$ and $a$ are constants, and $a > 0$

The curve passes through the points (1, 7) and (3, 175).

Calculate the value of $k$ and the value of $a$.

Sub values into $y=ka^x$

① : $7 = ka$

② : $175 = ka^3$

② ÷ ①

$$\frac{175 = ka^3}{7 = ka}$$

$25 = a^2$

$\sqrt{ANS}$

$a = \pm 5$

Given $a > 0$

$\therefore a = 5$

Sub $a=5$ into ①

$7 = ka$

$7 = k(5)$

$7 = 5k$

(÷5)

$\frac{7}{5} = k$

$k = \frac{7}{5}$

$a = 5$

**(Total 3 marks)**

**19.** **A** and **B** are straight lines.
Line **A** has equation $2y = 3x + 8$.
Line **B** goes through the points (–1, 2) and (2, 8).

Do lines **A** and **B** intersect?
You must show all your working.

| | |
|---|---|
| Equation of line B | $y=mx+c$ |
| $m=\frac{y_2-y_1}{x_2-x_1}$ | $m=\frac{8-2}{2-1}=\frac{6}{3}=2$ |
| Equation of line A | $2y=3x+8$ |
| $(\div 2)$ | $y=\frac{3}{2}x+4 \quad \therefore m=\frac{3}{2}$ |
| Conclusion | Since line A and line B have different gradients, they WILL intersect. |

**(Total 3 marks)**

Area of a Triangle SINE
Sine Rule

**20.**

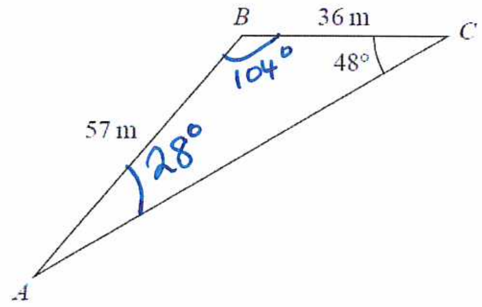



Diagram **NOT** accurately drawn

Work out the area of triangle *ABC*.
Give your answer correct to 3 significant figures.

$A = \frac{1}{2}ab\sin C$
... NEED $A\hat{B}C$
... Use Sine rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\therefore \frac{\sin 48}{57} = \frac{\sin \theta}{36}$

(×36) $36 \times \frac{\sin 48}{57} = \sin \theta$

*shift sin $^{-1}$ $28 = \theta$

Angles in triangle = 180° $180° - 28° - 48° = 104° = A\hat{B}C$

$A = \frac{1}{2}ab\sin C$: $A = \frac{1}{2}(57)(36)\sin(104)$

$A = 995.5234152$

$A = 996$ (3 s.f.)

996 ........................................................ $m^2$

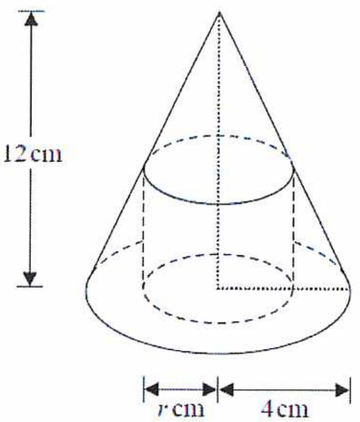
**(Total 4 marks)**

volume and surface area of complex shapes (cones / cylinders)

**21.** The diagram shows a cylinder inside a cone on a horizontal base.

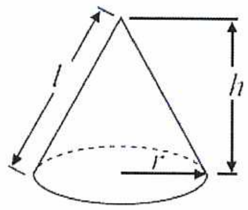
The cone and the cylinder have the same vertical axis.
The base of the cylinder lies on the base of the cone.

The circumference of the top face of the cylinder touches the curved surface of the cone.




**Volume of cone** $\frac{1}{3}\pi r^2 h$

**Curved surface area of cone** $= \pi r l$

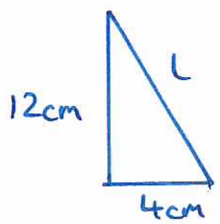



The height of the cone is 12 cm and the radius of the base of the cone is 4 cm.

(*a*) Work out the curved surface area of the cone.
Give your answer correct to 3 significant figures.

Curved surface area $= \pi r l$
$= 4\pi l$

NEED "l"...




Pythagoras: $a^2 + b^2 = c^2$
$12^2 + 4^2 = c^2$
$160 = c^2$
$\sqrt{160} = c = 12.649\ldots$

$\therefore$ curved surface area $= 4\pi(\sqrt{160})$
$= 158.95\ldots$
$= 159$ (3 s.f)

159 cm²

**(3)**

Let's work backwards...

The cylinder has radius $r$ cm and volume $V$ cm$^3$

(*b*) Show that $V = 12\pi r^2 - 3\pi r^3$

$V = \text{CSA} \times l$

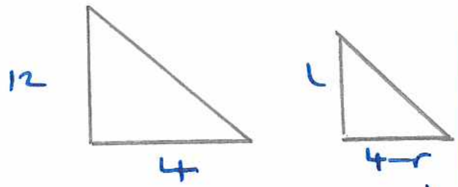
$= \pi r^2 \times \text{length}$

$V = 12\pi r^2 - 3\pi r^3$

$V = \pi r^2(12 - 3r)$

$\therefore$ we need to show that $l = 12 - 3r$

Similar shapes:




$\therefore \dfrac{4-r}{4} = \dfrac{l}{12}$

cross multiply

$12(4-r) = 4l$

$(\div 4)$

$3(4-r) = l$

expand

$\therefore 12 - 3r = l$, which is what we had to show

**(3)**

completed proof:

$V = \pi r^2(12-3r) = 12\pi r^2 - 3\pi r$

**(Total 6 marks)**

**TOTAL FOR PAPER IS 80 MARKS**