Higher tier unit 19a check in test

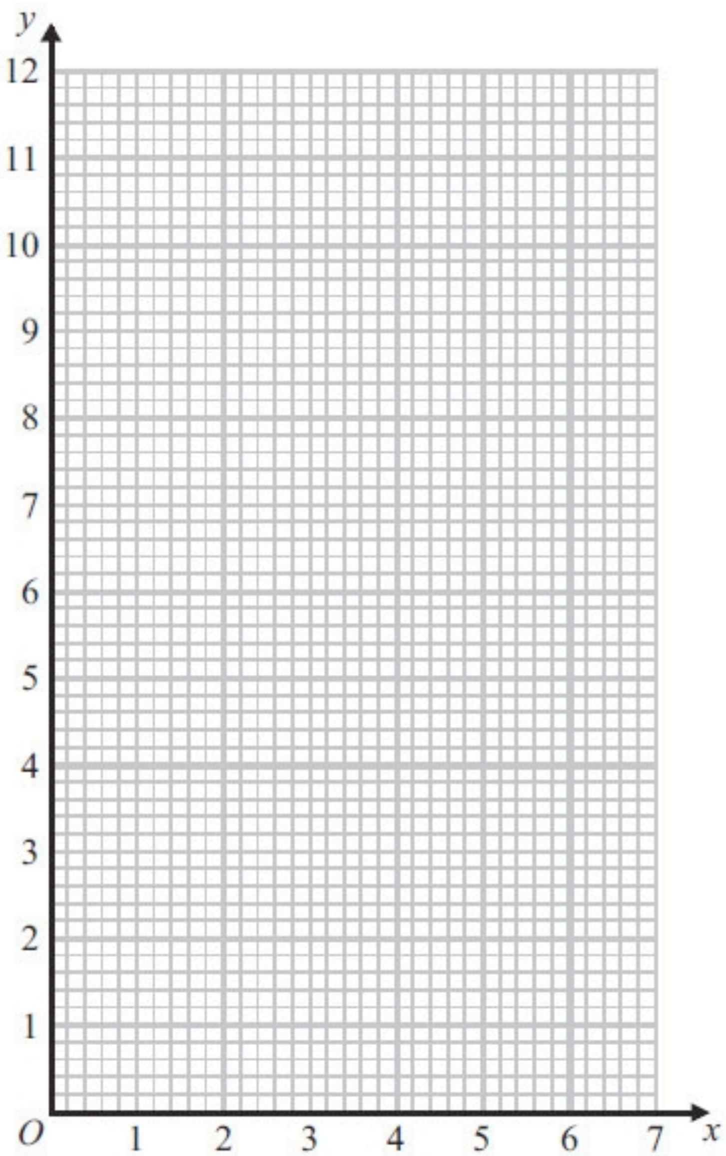
Calculator

[Q1–Q2 linked]

Q1. Complete the table of values for $y = \frac{6}{x}$

x	0.5	1	2	3	4	5	6
y		6	3		1.5		1

Q2. On the grid, draw the graph of $y = \frac{6}{x}$ for $0.5 \leq x \leq 6$

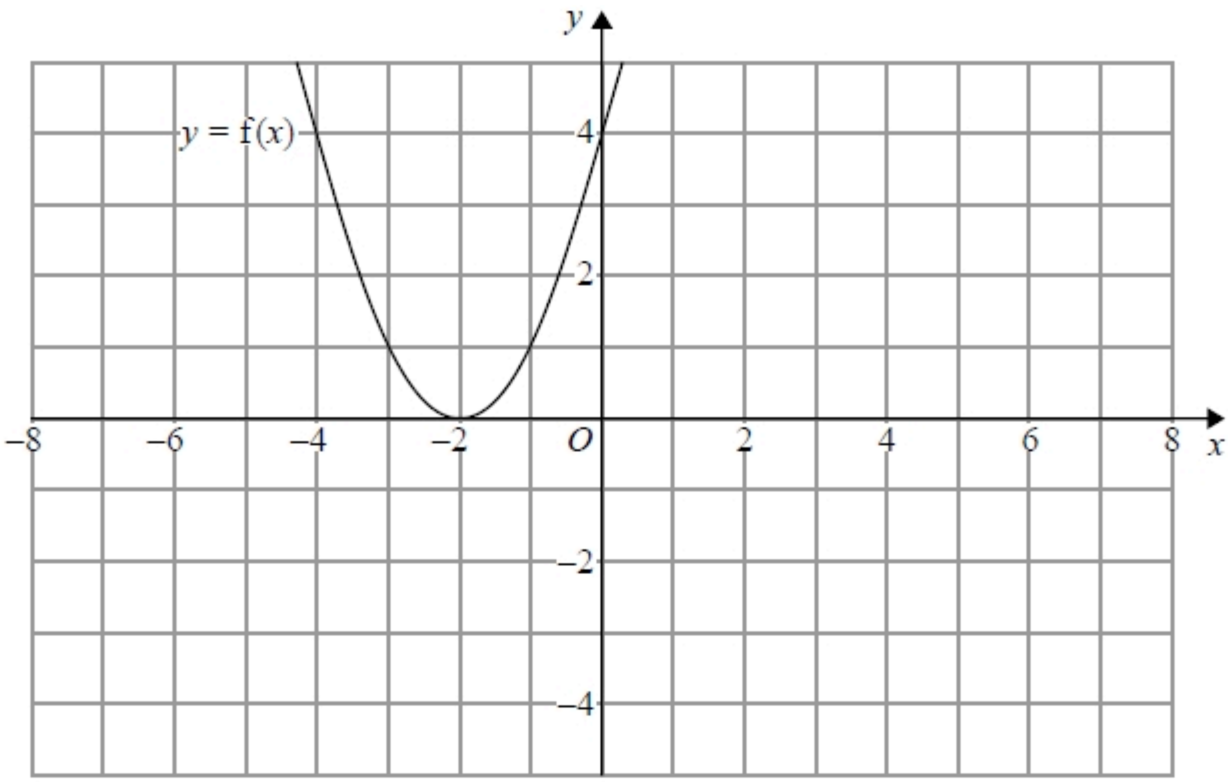


Q3. The graph of $y = \mathrm{f}(x)$ is transformed to give the graph of $y = -\mathrm{f}(x + 3)$

The point A on the graph of $y = \mathrm{f}(x)$ is mapped to the point P on the graph of $y = -\mathrm{f}(x + 3)$
The coordinates of point A are (9, 1)

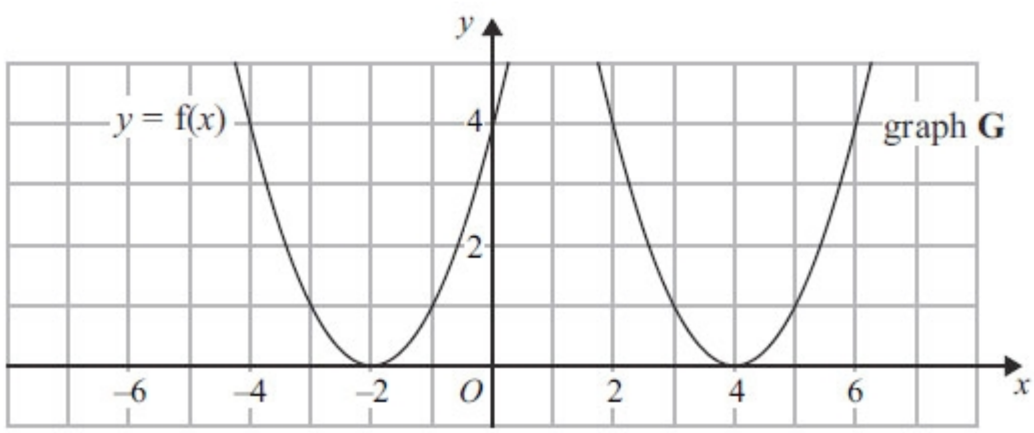
Find the coordinates of point P.

Q4. The graph of $y = \mathrm{f}(x)$ is shown on the grid.



Sketch the graph of $y = \mathrm{f}(-x)$

Q5. The graph of $y = \mathrm{f}(x)$ is shown on the grid.



The graph **G** is a translation of the graph of $y = \mathrm{f}(x)$.
Write down the equation of graph **G**.

Q6. The diagram shows a swimming pool in the shape of a prism.

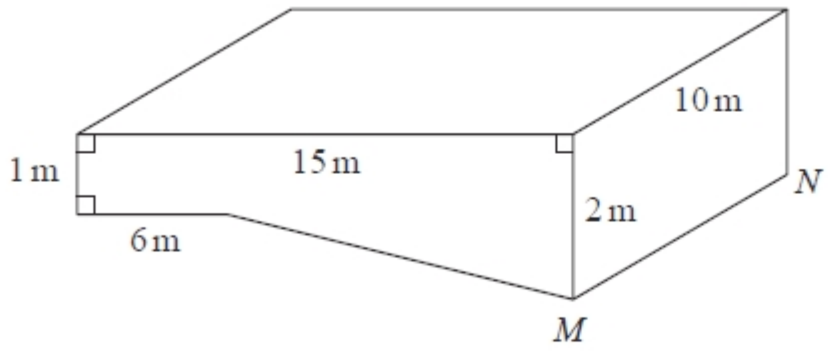
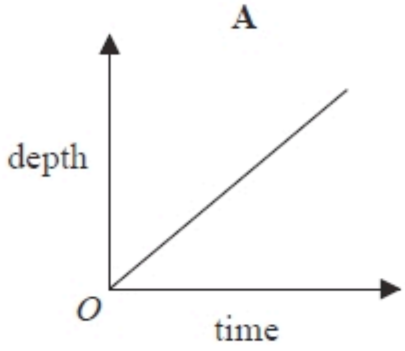


Diagram **NOT** accurately drawn

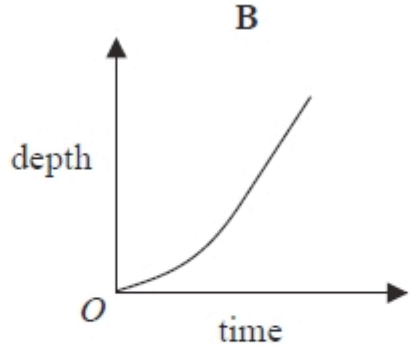
The swimming pool is empty.
The swimming pool is filled with water at a constant rate of 50 litres per minute.

Here are four graphs.

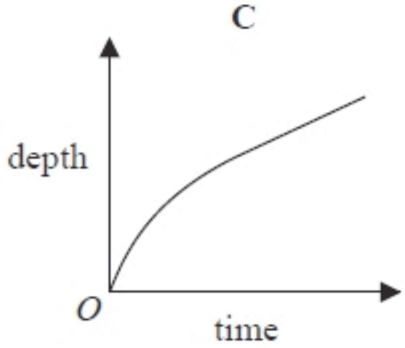
A



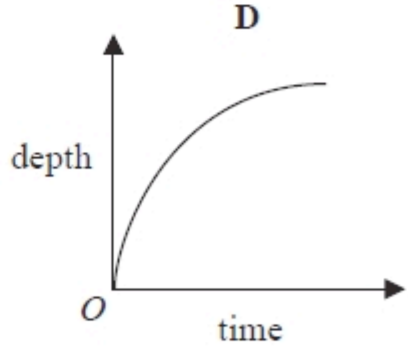
B



C



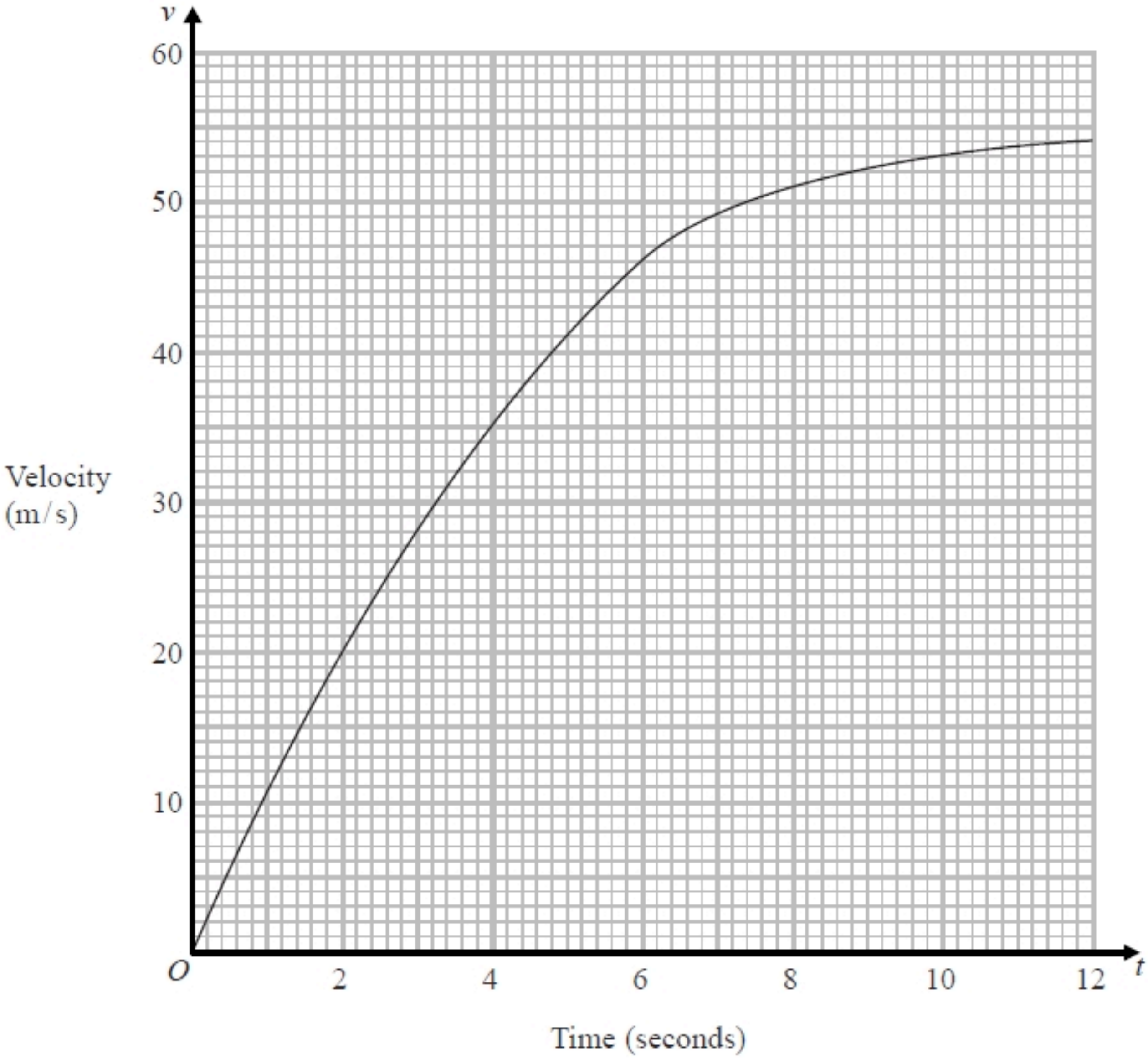
D



Write down the letter of the graph that best shows how the depth of the water in the pool above the line MN changes with time as the pool is filled.

[Q7–Q8 linked]

Q7. The graph shows information about the velocity, v m/s, of a parachutist t seconds after leaving a plane.



Work out an estimate for the acceleration of the parachutist at $t = 6$

Q8. Using the graph in question 7, work out an estimate for the distance fallen by the parachutist in the first 12 seconds after leaving the plane.
Use 3 strips of equal width.

[Q9–Q10 linked]

Q9. Louis and Robert are investigating the growth in the population of a type of bacteria.
They have two flasks A and B.

At the start of day 1, there are 1000 bacteria in flask A.
The population of bacteria grows exponentially at the rate of 50% per day.

The population of bacteria in flask A at the start of the 10th day is *k* times the population of bacteria in flask A at the start of the 6th day.

Find the value of *k*.

Q10. Louis and Robert are investigating the growth in the population of a type of bacteria.
They have two flasks A and B.

At the start of day 1, there are 1000 bacteria in flask A.
The population of bacteria grows exponentially at the rate of 50% per day.

At the start of day 1 there are 1000 bacteria in flask B.
The population of bacteria in flask B grows exponentially at the rate of 30% per day.

Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.

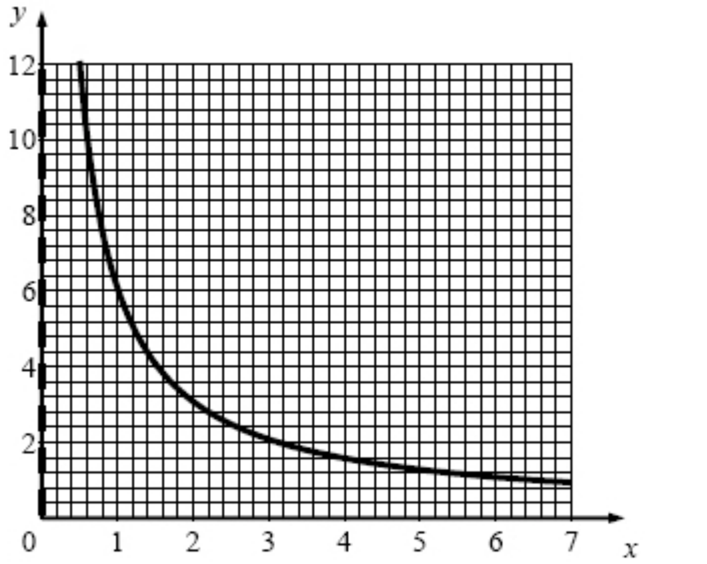
Topics listed in objectives

- Recognise, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$
- State the value of x for which the equation is not defined;
- Recognise, sketch and interpret graphs of exponential functions $y = k^x$ for positive values of k and integer values of x;
- Use calculators to explore exponential growth and decay;
- Set up, solve and interpret the answers in growth and decay problems;
- Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of $\mathrm{f}(x) + a$, or $\mathrm{f}(x - a)$:
    - apply to the graph of $y = \mathrm{f}(x)$ the transformations $y = -\mathrm{f}(x)$, $y = \mathrm{f}(-x)$ for linear, quadratic, cubic functions;
    - apply to the graph of y = f(x) the transformations $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ for linear, quadratic, cubic functions;
- Estimate area under a quadratic or other graph by dividing it into trapezia;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
    - for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
    - for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts;
- Interpret the area under a linear or non-linear graph in real-life contexts;
- Interpret the rate of change of graphs of containers filling and emptying;
- Interpret the rate of change of unit price in price graphs.

Answers

Q1. (0.5, 12), (3, 2), (5, 1.2)

Q2.



Q3. (6, –1)

Q4. Parabola passing through (0, 4), (2, 0), (4, 4)

Q5. $y = \mathrm{f}(x - 6)$

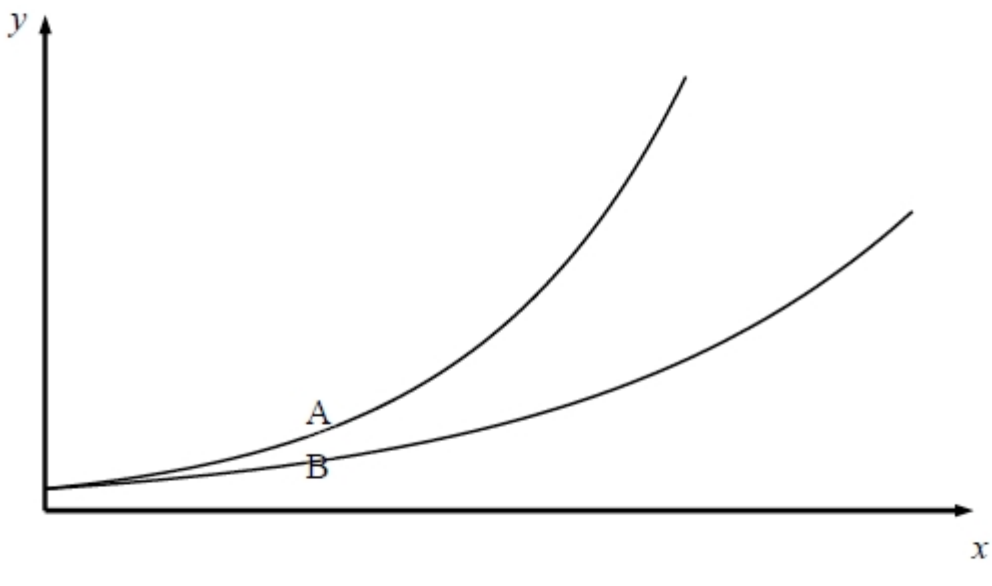
Q6. C

Q7. 3 to 4 m/s^2 (tangent at $t = 6$)

Q8. 452 m

Q9. $k = 1.5^4 = 5.0625$

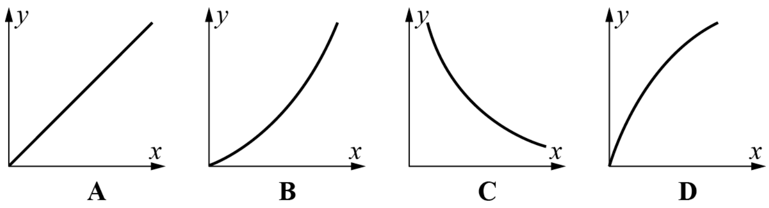
Q10.



Higher tier unit 19b check in test

Calculator

Q1. Write down the letter of the graph that shows variables in inverse proportion.



[Q2–3 linked]

Q2. y is directly proportional to the square of x.
Write a statement of proportionality to represent this relationship.

Q3. y is directly proportional to the square of x.
When $x = 3$, $y = 36$
Find the value of y when $x = 5$

Q4. d is inversely proportional to c
When $c = 280$, $d = 25$
Find the value of d when $c = 350$

Q5. In an experiment, values of f and g were taken.

f	3	6	8
g	54	432	1024

Which of these relationships fits the results?

$g \propto f^2$ $\quad$ $g \propto f^3$ $\quad$ $g \propto f$ $\quad$ $g \propto \sqrt{f}$

Q6. y is inversely proportional to x
When $x = 1.5$, $y = 36$
Find the value of y when $x = 6$

Q7. T is inversely proportional to d^2
$T = 160$ when $d = 8$
Find the value of T when $d = 0.5$

Q8. D is directly proportional to the cube of n.
Mary says that when n is doubled, the value of D is multiplied by 6
Mary is wrong.
Explain why.

Q9. A pendulum of length L cm has time period T seconds.
T is directly proportional to the square root of L.

The length of the pendulum is increased by 40%.
Work out the percentage increase in the time period.

Q10. When 20 litres of water are poured into any cylinder, the depth, D (in cm), of the water is inversely proportional to the square of the radius, r (in cm), of the cylinder.

When 20 litres of water is poured into a cylinder with radius 15 cm, the depth of the water is 28.4 cm.
When 20 litres of water is poured into another cylinder, the depth of the water is 64 cm.

Work out the radius of this cylinder, correct to 1 decimal place.

Topics listed in objectives

- Recognise and interpret graphs showing direct and inverse proportion;
- Identify direct proportion from a table of values, by comparing ratios of values, for x squared and x cubed relationships;
- Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity;
- Set up and use equations to solve word and other problems involving direct proportion;
- Use $y = kx$ to solve direct proportion problems, including questions where students find k, and then use k to find another value;
- Solve problems involving inverse proportion using graphs by plotting and reading values from graphs;
- Solve problems involving inverse proportionality;
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.

Answers

Q1. C
Q2. $y \propto x^2$
Q3. $y = 100$
Q4. $d = 20$
Q5. $g \propto f^3$
Q6. $y = 9$
Q7. $T = 40960$
Q8. The value of D should be multiplied by 8 (2^3 not 2×3)
Q9. 18.3%
Q10. 10.0 cm